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Notices

Edited by John W. Green and Gordon L. Walker

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MEETINGS

CALENDAR OF MEETINGS

Note: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meet- ing No.	Date	Place	Deadline for Abstracts*
588	February 22, 1962 (Emphasis on Finite Groups and Continuous Groups)	New York, New York	Jan. 9
589	April 12-14, 1962	Chicago, Illinois	Feb. 27
590	April 16-19, 1962	Atlantic City, New Jersey	Feb. 27
591	April 28, 1962	Monterey, California	Feb. 27
592	August 27-31, 1962 (67th Summer Meeting)	Vancouver, British Columbia	July 6
	January 21-25, 1963 (69th Annual Meeting)	Berkeley, California	
	August 26-30, 1963 (68th Summer Meeting)	Boulder, Colorado	
	January 21-24, 1964 (70th Annual Meeting)	Miami, Florida	
	August, 1964 (69th Summer Meeting)	Ann Arbor, Michigan	
	August, 1965	Ithaca, New York	
	August, 1966	New Brunswick, New Jersey	

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are January 9 and February 20.

The NOTICES of the American Mathematical Society is published by the Society seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is \$7.00. Price per copy, \$2.00. Special price for copies sold at registration desks of meetings of the Society, \$1.00 per copy. Subscriptions, orders for back numbers (none available before 1958), and inquiries should be addressed to the American Mathematical Society, 1350 Main Street, Ann Arbor, Michigan, or to 190 Hope Street, Providence 6, Rhode Island.

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SIXTY-EIGHTH ANNUAL MEETING

Sheraton-Gibson Hotel
Cincinnati, Ohio
January 22-25, 1962

PROGRAM

The sixty-eighth annual meeting of the American Mathematical Society will be held at Cincinnati, Ohio, in conjunction with the annual meeting of the Mathematical Association of America. All sessions will be held at the meeting headquarters, the Sheraton-Gibson Hotel.

The thirty-fifth Josiah Willard Gibbs Lecture will be delivered by Professor C. N. Yang at 8:00 P.M. on Monday, January 22, in the Roof Garden of the Sheraton-Gibson. Professor Yang will speak on "Symmetry principles in modern physics."

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, Professor Bertram Kostant will address the Society at 2:00 P.M. on Monday, January 22, in the Roof Garden. Professor Kostant will speak on "A survey of Lie group representation theory."

A special feature of this meeting is that Professors C. B. Morrey, Jürgen K. Moser, and Edward Nelson will address the Society on various topics in Partial Differential Equations, while Professor Bernard M. Dwork, Professor W. L. Chow, and Professor Oscar Zariski will speak on selected topics in Algebraic Geometry. The session on Partial Differential Equations will be held on Tuesday, January 23, from 10:00 A.M. to Noon in the Roof Garden with Professor Felix Browder presiding. The session on Algebraic Geometry will be held in the Roof Garden from 10:00 A.M. to Noon on Wednesday, January 24, with Professor John T. Tate as presiding officer.

The Cole Memorial Prize will be awarded in the Roof Garden at 2:00 P.M. on Tuesday, January 23.

In order to accommodate the Mathematical Association of America, the Annual Business Meeting of the Society will

be held at 1:00 P.M. on Wednesday, January 24, in the Roof Garden instead of 1:30 P.M. as previously announced. The following two items are to be considered:

According to the present By-laws the NOTICES are edited by the Executive Director. The Council recommended at Stillwater that the NOTICES be edited jointly by the Executive Director and the Secretary. The Society should discuss this matter since the recommendation of the Council calls for a change in the By-laws.

The Council also voted at Stillwater to delete the section of the NOTICES devoted to "Letters to the Editor." The Society at the Business Meeting in Stillwater voted that this item be put on the agenda for the meeting in Cincinnati.

The Council of the Society will meet in Parlor H on Tuesday, January 23, at 4:00 P.M.

The Employment Register will be located in Parlors O, P and Q, and will function on Tuesday, Wednesday, and Thursday from 9:00 A.M. to 5:00 P.M.

For the convenience of those attending the meetings, Parlors 4 and 5 on the Mezzanine Floor will be available as lounges from Monday to Friday except for Thursday evening.

ENTERTAINMENT AND RECREATION

At 6:00 P.M. on Wednesday, January 24, a bar will open in the Roof Garden of the Sheraton-Gibson Hotel to give members and their guests a chance to practice togetherness. Drinks will be sold.

At 7:00 P.M. following the cocktail hour, a complimentary dinner for 600 will be served in the Roof Garden for members and adult guests of the Society and the

Association. Tickets will be available at the registration desk on a first come, first served basis. There will be a service charge of \$1.50 each.

The dinner has been made possible through the generosity of the following contributors: The Avco Corporation, The Baldwin Piano Company, The Cincinnati Gas and Electric Company, The Cincinnati Milling Machine Company, The International Business Machines Corporation, The LeBlond Machine Tool Company, The Lunkenheimer Company, The Ohio National Life Insurance Company, The Taft Memorial Fund of the University of Cincinnati, The Union Central Life Insurance Company, The Western and Southern Life Insurance Company, and Xavier University.

The Taft Museum, a 19th century American home rich in historical background, containing a famous art collection, is within walking distance of the Sheraton-Gibson.

The Cincinnati Art Museum contains fifty galleries of painting, sculpture, china, silver, and pottery.

The Indoor Botanical Gardens exhibit tropical plants in waterfall settings.

The Natural History Museum has outstanding displays.

Other attractions will be publicized at Registration Headquarters.

REGISTRATION

Registration Headquarters will be located in the Ballroom Foyer of the Sheraton-Gibson. The office will be open on Sunday, January 21, from 2:00 P.M. to 8:00 P.M., Monday through Thursday from 9:00 A.M. to 5:00 P.M. and on Friday from 9:00 A.M. to 2:00 P.M. All those attending the meetings are requested to register at the Headquarters on arrival. A directory of registration and an information service will be maintained at Registration Headquarters.

The schedule for registration fees is as follows:

Members of participating organizations (except students)	\$2.00
First nonmember in member's family	.50

Other nonmembers in member's family	free
Students	free
Nonmembers not in any of the above categories	\$5.00

ACCOMMODATIONS

Accommodations are available at the Sheraton-Gibson with the overflow crowd being housed at the Netherland Hilton and the Sinton just around the corner. The reservation form on page 663 is to be employed. Rates are effectively the same at all these hotels: singles, \$8.50 and up; doubles and twins \$12.50 and up.

There are rooms for 25-50 students available at the YMCA at 1015 Elm Street at a cost of \$2.50 per day. Reservations should be made directly with the YMCA (attention Mr. Harry Strotham, Executive Secretary) and the Society should be mentioned.

TRAVEL INFORMATION

The Pennsylvania, Baltimore and Ohio, New York Central, Norfolk and Western, Chesapeake and Ohio, Louisville and Nashville, and Southern Railroads service the city. Airplanes to Cincinnati land at Boone County Airport in Kentucky about 12 miles from the hotel. Limousine service takes about 60 minutes and costs \$1.50. The Airlines are American, TWA, Delta, Eastern, Lake Central, and Piedmont.

COMMUNICATIONS

Mail and telegrams for those attending the meeting should be addressed in care of the American Mathematical Society, Hotel Sheraton-Gibson, Cincinnati, Ohio.

Committee on Arrangements:

- I. A. Barnett, Chairman
- H. L. Alder
- A. J. Macintyre
- W. E. Restemeyer
- J. W. T. Youngs

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes and no more. It is true that the contributed papers are scheduled at fifteen minute intervals, but the extra five minutes is for the purpose of permitting a listener to go from one session to another and to allow time for discussion of the preceding paper. The schedule in the Program must, therefore, be adhered to and to this end the time limit will be strictly enforced.

MONDAY, 9:00 A.M.

Session on Logic and Foundations, Parlors 7, 8 and 9, Mezzanine Floor

9:00 - 9:10

- (1) Elementary functionals and definition by recursion

Professor D. L. Kreider* and Dr. R. W. Ritchie, Dartmouth College (587-47)

9:15 - 9:25

- (2) Predictably computable functionals

Professor D. L. Kreider and Dr. R. W. Ritchie*, Dartmouth College (587-48)

9:30 - 9:40

- (3) On nested ordinal recursive functions and a subrecursive hierarchy

Mr. D. R. Anderson, Duke University (587-62)

9:45 - 9:55

- (4) Some observations on the axiom of choice

Dr. A. H. Kruse, New Mexico State University (587-27)

10:00 - 10:10

- (5) Nested pairs of well-formed occurrences in languages with quantifiers

Mr. David Schroer, The University of Rochester (587-44)

10:15 - 10:25

- (6) Degree of unsolvability and the rate of growth of functions

Mr. Stanley Tennenbaum, The University of Michigan (587-128)

(Introduced by Professor P. R. Halmos)

10:30 - 10:40

- (7) Extension of isomorphisms of polyadic algebras

Professor Aubert Daigneault, University of Montreal (587-136)

MONDAY, 9:00 A.M.

Session on Topology, Sheraton Room, Lower Lobby

9:00 - 9:10

- (8) An extension of a fine-cyclic element additivity theorem

Professor Russell Remage, Jr., University of Delaware (587-126)

9:15 - 9:25

- (9) A surface in S^3 is tame if it can be deformed into each complementary domain

Mr. J. P. Hempel, University of Wisconsin (587-45)

9:30 - 9:40

- (10) On subsets of a disk which contain all the wild points of the disk

Mr. R. J. Bean, University of Maryland (587-31)

(Introduced by Dr. Guydo Lehner)

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

- 9:45 - 9:55
 (11) A sufficient condition for the imbedding of Peano continua in 1-manifolds
 Dr. Beauregard Stubblefield, Michigan State University Oakland (587-100)
- 10:00 - 10:10
 (12) Factorization of cells
 Professor K. W. Kwun, Seoul National University, Korea and University
 of Wisconsin and Professor F. A. Raymond*, University of Wisconsin
 (587-71)
- 10:15 - 10:25
 (13) A decomposition theorem for E^4
 Professor M. E. Hamstrom, University of Illinois (587-38)
- 10:30 - 10:40
 (14) Some almost polyhedral wild arcs
 Professor B. J. Ball, University of Georgia and Mr. W. R. Alford*, Tulane
 University (587-127)
- 10:45 - 10:55
 (15) Almost locally polyhedral curves in E^n
 Mr. J. C. Cantrell*, University of Tennessee and Mr. C. H. Edwards, Jr.,
 University of Wisconsin (587-13)

MONDAY, 9:00 A.M.

Session on Analysis, Gibson Room, Lower Lobby

- 9:00 - 9:10
 (16) A complete set of unitary invariants for operators generating finite
 W^* -algebras of type I.
 Dr. C. M. Pearcy, Humble Oil and Refining Company, Houston, Texas
 (587-42)
- 9:15 - 9:25
 (17) An extension of the nonhomogeneous Farkas theorem
 Professor C. C. Braunschweiger, University of Delaware (587-20)
- 9:30 - 9:40
 (18) A note on the Riesz representation theorem
 Professor D. H. Tucker, University of Utah (587-16)
- 9:45 - 9:55
 (19) Geometric structure of conditional basis systems in a Banach space
 Professor R. E. Fullerton, University of Maryland (587-82)
- 10:00 - 10:10
 (20) Interpolation spaces
 Professor E. Gagliardo, University of Genoa and University of Kansas
 (587-117)
 (Introduced by Professor Aronszajn)
- 10:15 - 10:25
 (21) Convergence on filters
 Professor J. W. Brace, University of Maryland (587-32)
- 10:30 - 10:40
 (22) Convolution of sequences
 Professor G. U. Brauer, University of Minnesota (587-18)
- 10:45 - 10:55
 (23) Classes of orthogonal polynomials with totally positive moment sequences
 Dr. Herman Van Rossum, Michigan State University (587-103)
- 11:00 - 11:10
 (24) A property of orthogonal polynomials
 Professor J. J. Price, Cornell University (587-29)

MONDAY, 2:00 P.M. - 3:00 P.M.

Invited Address, Roof Garden

A survey of Lie group representation theory

Professor Bertram Kostant, University of California, Berkeley

MONDAY, 3:30 P.M.

Session on Geometry, Parlors 7, 8 and 9, Mezzanine Floor

3:30 - 3:40

- (25) The extension of minimal surfaces intersected in convex curves by parallel planes

Professor J. C. C. Nitsche, University of Minnesota (587-55)

3:45 - 3:55

- (26) Lattice coverings of n-dimensional euclidean space by spheres

Dr. M. N. Bleicher, University of California, Berkeley (587-114)

4:00 - 4:10

- (27) Parallel and central transformations of Riemannian manifolds

Professor C. C. Hsiung and Dr. H. I. Nassar*, Lehigh University (587-39)

4:15 - 4:25

- (28) Some complex integration problems

Professor Albert Nijenhuis, University of Washington and Institute for Advanced Study, Princeton, New Jersey and Professor W. B. Woolf*, University of Washington (587-53)

4:30 - 4:40

- (29) Complementary manifolds in matrix algebras

Professor Andrew Sobczyk, University of Miami (587-64)

4:45 - 4:55

- (30) Does the surface-measure of a sphere depend on its location in space?

Professor B. A. Fusaro, University of Oklahoma (587-92)

5:00 - 5:10

- (31) Graphical solution of problems involving the incidence of points, lines and planes

Professor Aboulghassem Zirakzadeh, University of Colorado (587-123)

MONDAY, 3:30 P.M.

Session on Statistics and Probability, Sheraton Room, Lower Lobby

3:30 - 3:40

- (32) An entrance boundary for Markov processes

Professor F. B. Knight, University of Minnesota (587-129)

3:45 - 3:55

- (33) Markov processes equivalent under a change of time

Professor R. M. Blumenthal, Institute for Advanced Study and Professor R. K. Gettoor*, University of Washington (587-80)

4:00 - 4:10

- (34) Representation of an isotropic diffusion as a skew product

Dr. A. R. Galmarino, Northeastern University (587-63)

4:15 - 4:25

- (35) A zero-one property of mixing sequences of events

Professor Louis Sucheston, University of Wisconsin-Milwaukee (587-102)

4:30 - 4:40

- (36) On decomposition into independent components

Dr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey (587-41)

4:45 - 4:55

- (37) On regression, projection, and conditional expectation. Preliminary report
Professor H. D. Brunk, University of California, Riverside (587-115)

5:00 - 5:10

- (38) An estimate of the compounding distribution of a compound Poisson distribution

Professor H. G. Tucker, University of California, Riverside (587-6)

5:15 - 5:25

- (39) On the type of an entire characteristic function of finite order

Professor B. Ramachandran, The Catholic University of America (587-46)
(Introduced by Professor Eugene Lukacs)

MONDAY, 3:30 P.M.

Session on Analysis, Gibson Room, Lower Lobby

3:30 - 3:40

- (40) On order and commutativity in Jordan operator algebras. Preliminary report
Mr. D. M. Topping, Tulane University (587-25)

3:45 - 3:55

- (41) Normalcy in operator algebras

Professor R. V. Kadison, Columbia University (587-119)

4:00 - 4:10

- (42) Trace formulas for a perturbed operator

Professor R. C. Gilbert* and Professor V. A. Kramer, University of California, Riverside (587-73)

4:15 - 4:25

- (43) The spectra of minimal self-adjoint extensions of a symmetric operator

Professor R. W. McKelvey, University of Colorado (587-98)

4:30 - 4:40

- (44) On normal dilations and spectral sets. Preliminary report

Mr. Arnold Lebow, University of Michigan (587-11)

4:45 - 4:55

- (45) A Šilov boundary for subregular functions. Preliminary report

Professor L. L. Helms, University of Illinois (587-7)

5:00 - 5:10

- (46) Closed linear operators and associated continuous linear operators

Dr. Seymour Goldberg, New Mexico State University (587-23)

5:15 - 5:25

- (47) A new classification of partially hypoelliptic differential operators

Mr. Jöran Friberg, Yale University (587-77)

5:30 - 5:40

- (48) Integration of the first order differential inequality with distributions

Dr. W. Mlak, University of Maryland (587-17)

(Introduced by R. E. Fullerton)

MONDAY, 8:00 P.M. - 9:00 P.M.

Josiah Willard Gibbs Lecture, Roof Garden

Symmetry principles in modern physics

Professor C. N. Yang, Institute for Advanced Study

TUESDAY, 10:00 A.M. - NOON

Lectures on Topics in Partial Differential Equations, Roof Garden

Presiding Officer: Professor Felix Browder, Yale University and
Massachusetts Institute of Technology

Some recent developments in partial differential equations
Professor C. B. Morrey, Jr., University of California, Berkeley

An implicit function theorem and its application to differential equations
Professor Jürgen K. Moser, Massachusetts Institute of Technology

Potential theory and partial differential equations
Professor Edward Nelson, Princeton University

TUESDAY, 2:00 P.M. - 3:00 P.M.

Awarding of the Cole Memorial Prize, Roof Garden

TUESDAY, 3:30 P.M.

Session on Algebra, Parlors 7, 8 and 9, Mezzanine Floor

3:30 - 3:40

(49) On the splitting of Abelian groups

Professor E. A. Walker, Professor J. M. Irwin* and Miss Carol Peercy,
New Mexico State University (587-72)

3:45 - 3:55

(50) A theorem of finite group algebras

Mr. D. B. Coleman, Vanderbilt University (587-116)

4:00 - 4:10

(51) Relative difference sets

Professor A. T. Butson, University of Miami (587-112)

4:15 - 4:25

(52) An analogue to the Witt identity

Professor Seymour Sherman, Wayne State University (587-120)

4:30 - 4:40

(53) On the uniqueness of character semigroups

Professor N. J. Rothman, University of Rochester (587-54)

4:45 - 4:55

(54) Factorization in noncommutative power series rings

Professor P. M. Cohn, Yale University (587-61)

(Introduced by Nathan Jacobson)

5:00 - 5:10

(55) Johnson and Goldie prime rings

Professor Carl Faith, Institute for Advanced Study and Pennsylvania
State University (587-81)

5:15 - 5:25

(56) Super-atomic Boolean algebras. Part II. Preliminary report

Mr. G. W. Day, Purdue University (587-107)

5:30 - 5:40

(57) Jordan-algebras and their applications to analysis

Professor Max Koecher, University of Minnesota (587-124)

(Introduced by Professor H. Röhrli)

TUESDAY, 3:30 P.M.

Session on Applied Mathematics, Sheraton Room, Lower Lobby

3:30 - 3:40

(58) Integration on quasigroups. Preliminary report

Mr. T. S. Frank, Syracuse University (587-10)

3:45 - 3:55

(59) The relation between continuity and differentiability of functions on algebras

Professor J. C. Wilson, Florida Presbyterian College and Professor
R. F. Rinehart*, Case Institute of Technology (587-52)

- 4:00 - 4:10
 (60) A Daniell's approach to integration on locally compact spaces
 Professor Choy-tak Taam, Georgetown University (587-50)
- 4:15 - 4:25
 (61) A sufficient condition for total monotonicity
 Professor B. E. Rhoades, Lafayette College (587-9)
- 4:30 - 4:40
 (62) Approximation by polynomials with positive coefficients
 Professor G. G. Lorentz, Syracuse University (587-40)
- 4:45 - 4:55
 (63) Constructions of subadditive functions
 Mr. R. G. Laatsch, Oklahoma State University (587-28)
- 5:00 - 5:10
 (64) An integral representation theorem for functions convex with respect to a pair of functions
 Mr. Larry Armijo, Rice University (587-108)
 (Introduced by Mr. Jim Douglas, Jr.)
- 5:15 - 5:25
 (65) A generalization of the Stieltjes transform. Preliminary report
 Mr. H. C. Miller, Jr., University of Alabama (587-101)
- 5:30 - 5:40
 (66) Difference kernels
 Mr. Marvin Shinbrot, The University of Chicago (587-122)

TUESDAY, 3:30 P.M.

Session on Analysis, Gibson Room, Lower Lobby

- 3:30 - 3:40
 (67) Oscillations of a pendulum under parametric excitation
 Professor R. A. Struble, North Carolina State College (587-19)
- 3:45 - 3:55
 (68) On the existence of a solution to a nonlinear wave equation with mixed boundary conditions
 Mr. H. L. Johnson, Wright-Patterson Air Force Base, Dayton, Ohio (587-105)
- 4:00 - 4:10
 (69) Characteristics of a rotating fluid
 Mr. H. C. Sebring, General Electric Company, Syracuse, New York (587-8)
 (Introduced by J. W. T. Youngs)
- 4:15 - 4:25
 (70) A numerical integration of the Navier-Stokes equations
 Professor J. Gillis*, Institute for Space Studies, New York, New York, and Mr. M. Shimshoni, Weizmann Institute of Science, Israel (587-59)
- 4:30 - 4:40
 (71) Quasi-sinusoidal solutions of nonlinear differential-difference equations
 Dr. G. S. Jones, RIAS, Baltimore, Maryland (587-94)
- 4:45 - 4:55
 (72) On the boundary value problem of perfect reflection in the theory of electromagnetic wave fields
 Dr. Peter Werner, Mathematics Research Center, University of Wisconsin (587-5)
 (Introduced by Professor R. E. Langer)
- 5:00 - 5:10
 (73) Coalition bargaining in N-person games
 Professor E. D. Nering, Arizona State University (587-113)

5:15 - 5:25

- (74) Finding zeros of general functions automatically

Dr. M. A. Hyman, IBM Systems Center, Bethesda, Maryland (587-68)

5:30 - 5:40

- (75) An iterative procedure for finding the zeros of a polynomial

Professor J. E. Maxfield, University of Florida (587-58)

WEDNESDAY, 10:00 A.M. - NOON

Lectures on Topics in Algebraic Geometry, Roof Garden

Presiding Officer: Professor John T. Tate, Harvard University

On the zeta function of an algebraic variety

Professor Bernard M. Dwork, Institute for Advanced Study

Intersection theory in algebraic geometry

Professor W. L. Chow, Johns Hopkins University

Some remarks on the present state of the problem of reduction of singularities,

Professor Oscar Zariski, Harvard University

WEDNESDAY, 1:00 P.M.

Business Meeting, Roof Garden

WEDNESDAY, 3:00 P.M.

Session on Topology, Parlors 7, 8 and 9, Mezzanine Floor

3:00 - 3:10

- (76) On the braid groups of the projective plane

Mr. James Van Buskirk, University of Wisconsin (587-125)

3:15 - 3:25

- (77) A note on compact connected semigroups

Mr. John Selden, University of Georgia (587-65)

3:30 - 3:40

- (78) Steenrod reduced powers in Ext. Preliminary report

Professor D. B. A. Epstein, Institute for Advanced Study (587-56)

3:45 - 3:55

- (79) Piecewise linear real functions on combinatorial manifolds

Professor A. A. Kosinski, University of California, Berkeley (587-79)

4:00 - 4:10

- (80) The stable symplectic group and the cohomology of an algebra

Dr. A. L. Liulevicius, The University of Chicago and The Institute for Advanced Study (587-84)

4:15 - 4:25

- (81) A note on almost periodic functions. Preliminary report

Mr. Ta-Sun Wu, Tulane University (587-30)

4:30 - 4:40

- (82) Proximity relations in transformation groups

Dr. J. P. Clay, Remington Rand UNIVAC, Philadelphia, Pennsylvania (587-111)

4:45 - 4:55

- (83) Jacobians and symmetric products

Professor Rolph Schwarzenberger, The Institute for Advanced Study (587-24)

Session on Integral and Differential Equations, Sheraton Room, Lower Lobby

3:00 - 3:10

- (84) The product integral solution of an integral equation. Preliminary report
Professor J. R. Dorroh, The University of Texas (587-35)

3:15 - 3:25

- (85) Integral equations and semi-groups
Professor J. S. MacNerney, University of North Carolina (587-2)

3:30 - 3:40

- (86) Solutions of certain linear homogeneous integral equations. Preliminary report
Professor D. G. Dickson, The University of Michigan (587-34)

3:45 - 3:55

- (87) On the solution of certain dual integral equations
Professor J. Burlak, North Carolina State College (587-90)

4:00 - 4:10

- (88) Ordinary differential equations determined by contracting vector fields.
Preliminary report
Mr. J. V. Ryff, Stanford University (587-69)

4:15 - 4:25

- (89) On first order differential equations
Professor F. G. Brauer, Mathematics Research Center, U. S. Army and
University of Wisconsin (587-22)

4:30 - 4:40

- (90) Solutions of first order differential equations, including singular cases.
Preliminary report
Professor H. G. Ellis, University of Utah (587-36)

4:45 - 4:55

- (91) On the stability of certain systems of parabolic difference equations
Professor D. G. Aronson, University of Minnesota and Stanford University (587-75)

5:00 - 5:10

- (92) A bound for solutions of uniformly elliptic equations
Professor H. F. Weinberger, Institute of Technology, University of
Minnesota (587-21)

5:15 - 5:25

- (93) Stability, uniform stability and Lyapunov's second method. Preliminary report
Mr. G. R. Sell, University of Michigan (587-12)

WEDNESDAY, 3:00 P.M.

Session on Analysis, Gibson Room, Lower Lobby

3:00 - 3:10

- (94) Existence of interpolating functions of exponential type
Professor R. F. DeMar, Miami University (587-91)

3:15 - 3:25

- (95) The real zeros of three successive derivatives of an entire function
Dr. Simon Hellerstein, Stanford University (587-93)

3:30 - 3:40

- (96) A note on entire functions and a conjecture of Erdős
Mr. Alfred Gray, University of California at Los Angeles and Professor
S. M. Shah*, University of Kansas (587-15)

3:45 - 3:55

- (97) The zeros of the partial sums of certain small entire functions
Dr. G. W. Hedstrom*, University of Michigan and Professor Jacob
Korevaar, Stanford University (587-118)

- 4:00 - 4:10
 (98) Summability of ordinary Dirichlet series by Perron-type matrices
 Dr. W. T. Sledd, Michigan State University (587-87)
- 4:15 - 4:25
 (99) Order of functions holomorphic in the unit disk
 Mr. D. C. Rung, Pennsylvania State University (587-76)
- 4:30 - 4:40
 (100) The Lebesgue constants for regular Taylor and $[F, d_n]$ summation of Fourier series
 Professor Lee Lorch*, University of Alberta, Canada and Professor D. J. Newman, Yeshiva University (587-4)
- 4:45 - 4:55
 (101) The summability of series of Hermite polynomials
 Professor G. G. Bilodeau, Boston College (587-74)
- 5:00 - 5:10
 (102) On equivalence theorems of the Knopp-Schnee-Hausdorff type
 Professor W. B. Jurkat, Syracuse University and Professor Alexander Peyerimhoff*, University of Utah (587-67)

THURSDAY, 2:00 P.M.

Session on Topology, Parlors 7, 8 and 9, Mezzanine Floor

- 2:00 - 2:10
 (103) Extended topology: Generation of subadditive and additive subfunctions
 Professor P. C. Hammer, University of Wisconsin (587-83)
- 2:15 - 2:25
 (104) Zero-sets in topological spaces
 Professor Costas Kassimatis, North Carolina State College (587-110)
- 2:30 - 2:40
 (105) On the representation of a manifold as a singular cell
 Professor M. Brown, Institute for Advanced Study, and Mr. B. Cassler*, University of Wisconsin (587-106)
- 2:45 - 2:55
 (106) Homogeneity of infinite products of manifolds with boundary
 Professor M. K. Fort, Jr., University of Georgia (587-51)
- 3:00 - 3:10
 (107) Additivity of topological properties
 Mr. David Ryeburn, Ohio State University (587-86)
- 3:15 - 3:25
 (108) ϵ -mappings onto manifolds. Preliminary report
 Professor Sibe Mardešić, University of Zagreb, Yugoslavia, and University of Washington and Professor Jack Segal*, University of Washington (587-109)
- 3:30 - 3:40
 (109) Endomorphisms of minimal sets. Preliminary report
 Dr. Joseph Auslander, RIAS, Baltimore, Maryland (587-89)
- 3:45 - 3:55
 (110) Some properties of minimal topological spaces
 Dr. M. P. Berri, University of California at Los Angeles (587-26)

THURSDAY, 2:00 P.M.

Session on Algebra, Sheraton Room, Lower Lobby

- 2:00 - 2:10
 (111) Fermat numbers and perfect numbers
 Mr. Alexander Hurwitz, University of California, Los Angeles and Dr. J. L. Selfridge*, University of Washington (587-104)

- 2:15 - 2:25
 (112) On some connections between Mertens' hypothesis and Dirichlet's L-series
 Dr. H. E. Richert, Syracuse University (587-88)
- 2:30 - 2:40
 (113) r as an r th power residue
 Professor J. B. Muskat, University of Pittsburgh (587-96)
- 2:45 - 2:55
 (114) On mean values of trigonometrical sums in algebraic number fields
 Dr. Otto Koerner, University of Utah (587-66)
 (Introduced by Professor Alexander Peyerimhoff)
- 3:00 - 3:10
 (115) On the square of a polynomial
 Professor L. Moser* and Mr. J. R. Pounder, University of Alberta
 (587-43)
- 3:15 - 3:25
 (116) The permanent of a symmetric matrix
 Professor Marvin Marcus*, University of British Columbia and Professor
 Morris Newman, United States National Bureau of Standards, Washington,
 D. C. (587-85)
- 3:30 - 3:40
 (117) Permutation polynomials over a finite field
 Mr. R. M. McConnel, Duke University (587-78)
- 3:45 - 3:55
 (118) On the commutator subgroup of the orthogonal group over the 2-adic numbers
 Professor Barth Pollak, Institute for Defense Analyses, Princeton, New
 Jersey (587-99)
- 4:00 - 4:10
 (119) Disjoint pairs of sets and incidence matrices
 Professor Marvin Marcus, University of British Columbia and Professor
 Henryk Minc*, University of Florida (587-57)
- 4:15 - 4:25
 (120) Distance to nearest integer of powers of rational numbers
 Professor A. S. Fraenkel, University of Oregon (587-37)

THURSDAY, 2:00 P.M.

Session on Analysis, Gibson Room, Lower Lobby

- 2:00 - 2:10
 (121) Darboux functions of Baire class one and derivatives
 Dr. C. J. Neugebauer, Purdue University (587-97)
- 2:15 - 2:25
 (122) Entropies and n th widths of several sets of functions. Preliminary report
 Mr. G. F. Clements, Syracuse University (587-3)
- 2:30 - 2:40
 (123) Variational problems of minimal surface type
 Professor H. B. Jenkins* and Professor J. B. Serrin, University of
 Minnesota (587-95)
- 2:45 - 2:55
 (124) On the Wiener-Perron method for the Dirichlet problem. Preliminary report
 Professor E. C. Schlesinger, Wesleyan University (587-49)
- 3:00 - 3:10
 (125) Interpolation by harmonic polynomials
 Professor J. H. Curtiss, University of Miami (587-70)
- 3:15 - 3:25
 (126) Properties of harmonic functions of three real variables given by the
 Bergman-Whittaker operator. I
 Professor J. M. Mitchell, Pennsylvania State University (587-33)

3:30 - 3:40

- (127) The Šilov boundary of a subalgebra of measures on a group
Professor A. B. Simon, Northwestern University (587-121)

3:45 - 3:55

- (128) Measure preserving functions on locally compact spaces
Professor P. T. Church*, Institute for Advanced Study and Professor
M. E. Mahowald, Syracuse University (587-60)

4:00 - 4:10

- (129) On a technique of proving ergodic theorem in particular cases
Professor M. Z. v. Krzywoblocki, Michigan State University (587-1)

Bloomington, Indiana

J. W. T. Youngs
Associate Secretary

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TIME TABLE
(Eastern Standard Time)

SUNDAY, January 21	American Mathematical Society	Mathematical Association of America
2:00 p.m. - 8:00 p.m.	REGISTRATION - BALLROOM FOYER	
MONDAY, January 22		
9:00 a.m. - 5:00 p.m.	REGISTRATION - BALLROOM FOYER	
9:00 a.m.	Session on Logic and Foundation - Parlors 7, 8, 9 Mezzanine Floor	
9:00 a.m.	Session on Topology - Sheraton Room, Lower Lobby	
9:00 a.m.	Session on Analysis - Gibson Room, Lower Lobby	
2:00 p.m.	Invited Address - Bertram Kostant Roof Garden	
3:30 p.m.	Session on Geometry - Parlors 7, 8, 9 Mezzanine Floor	
3:30 p.m.	Session on Statistics and Probability - Sheraton Room, Lower Lobby	
3:30 p.m.	Session on Analysis - Gibson Room, Lower Lobby	
8:00 p.m.	Josiah Willard Gibbs Lecture - C.N. Yang Roof Garden	
TUESDAY, January 23		
9:00 a.m. - 5:00 p.m.	REGISTRATION - BALLROOM FOYER EMPLOYMENT REGISTER - PARLORS O, P, Q EXHIBITS - BALLROOM	
10:00 a.m.	Lectures: Partial Differential Equations C.B. Morrey, Jr. Jürgen K. Moser Edward Nelson Roof Garden	
2:00 p.m.	Awarding of the Cole Memorial Prize - Roof Garden	Executive and Finance Committees - Parlor I
3:30 p.m.	Session on Algebra - Parlors 7, 8, 9 Mezzanine Floor	
3:30 p.m.	Session on Applied Mathematics - Sheraton Room, Lower Lobby	
3:30 p.m.	Session on Analysis - Gibson Room, Lower Lobby	
4:00 p.m.	Council Meeting - Parlor H	

TIME TABLE
(Eastern Standard Time)

WEDNESDAY, January 24	American Mathematical Society	Mathematical Association of America
9:00 a. m. - 5:00 p. m.	REGISTRATION - BALLROOM FOYER EMPLOYMENT REGISTER - PARLORS O, P, Q EXHIBITS - BALLROOM	
8:45 a. m.		Board of Governors - Parlor I
10:00 a. m.	Lectures: Algebraic Geometry Bernard M. Dwork W. L. Chow Oscar Zariski - Roof Garden	
1:00 p. m.	Business Meeting - Roof Garden	
2:30 p. m.		First Session: Logic - L. A. Henkin Roof Garden
3:00 p. m.	Session on Topology - Parlors 7, 8, 9, Mezzanine Floor	
3:00 p. m.	Session on Integral and Differential Equations - Sheraton Room, Lower Lobby	
3:00 p. m.	Session on Analysis - Gibson Room, Lower Lobby	
3:30 p. m.		First Session: Logic - Hartley Rogers Roof Garden
4:30 p. m.		First Session: Logic - F. B. Fitch Roof Garden
6:00 p. m.	COCKTAIL HOUR - ROOF GARDEN	
7:00 p. m.	DINNER - ROOF GARDEN	
THURSDAY, January 25		
9:00 a. m. - 5:00 p. m.	REGISTRATION - BALLROOM FOYER EMPLOYMENT REGISTER - PARLORS O, P, Q EXHIBITS - BALLROOM	
9:00 a. m.		Second Session: Real Analysis L. W. Cohen - Roof Garden
9:25 a. m.		Second Session: Real Analysis N. E. Steenrod - Roof Garden
10:00 a. m.		Second Session: Real Analysis B. J. Pettis - Roof Garden
10:35 a. m.		Business Meeting - Roof Garden
1:00 p. m.	CONFERENCE BOARD MEETING - PARLOR H	
2:00 p. m.	Session on Topology - Parlors 7, 8, 9, Mezzanine Floor	
2:00 p. m.	Session on Algebra - Sheraton Room, Lower Lobby	
2:00 p. m.	Session on Analysis - Gibson Room, Lower Lobby	

TIME TABLE
(Eastern Standard Time)

FRIDAY, January 26	American Mathematical Society	Mathematical Association of America
9:00 a. m. - 2:00 p. m.	REGISTRATION - BALLROOM FOYER	
8:30 a. m.	REPORT FROM THE CONFERENCE BOARD OF THE MATHEMATICAL SCIENCES - ROOF GARDEN	
9:30 a. m.		Third Session: Geometry Ernst Snapper - Roof Garden
10:30 a. m.		Third Session: Geometry Marshall Hall, Jr. - Roof Garden
11:30 a. m.		Address: C. B. Allendoerfer Roof Garden
2:00 p. m.		Fourth Session: Geometry Arno Jaeger - Roof Garden
3:00 p. m.		Fourth Session: Geometry H. S. M. Coxeter - Roof Garden
4:00 p. m.		Fourth Session: Geometry Howard Levi - Roof Garden

NAMES TO LOOK FOR IN MATHEMATICS

ARNO JAEGER: University of Cincinnati. Jaeger's *INTRODUCTION TO ANALYTIC GEOMETRY AND LINEAR ALGEBRA* "is an unusual book," wrote E. J. F. Primrose in *The Mathematic Gazette*. "The author's object is to develop linear algebra and geometry together, so that the student can see the motive for introducing the various algebraic concepts, and so that the abstract algebraic ideas can be illustrated by more concrete geometrical examples. The main theme of the book is that of vectors . . . if (a student) mastered it he would have at his disposal some of the most important ideas and techniques of modern mathematics." 1960, 305 pp., \$6.00

HOLT, RINEHART and WINSTON

- RALPH B. CROUCH and ELBERT A. WALKER, both of New Mexico State University
INTRODUCTION TO MODERN ALGEBRA AND ANALYSIS
Spring 1962, 256 pp., \$7.00 tent.
- RICHARD E. JOHNSON, University of Rochester; NEAL H. MCCOY, Smith College; ANNE F. O'NEILL, Wheaton College
INTRODUCTION TO MATHEMATICAL ANALYSIS
1962, 384 pp., \$7.00 tent.
- WILHELM MAAK, Mathematical Institute of the University, Göttingen, Germany; GEORGE STRIKER, translator
AN INTRODUCTION TO MODERN CALCULUS
March 1962, 400 pp., \$7.00 tent.
- LOUIS L. PENNISI, Univ. of Ill.
ELEMENTS OF COMPLEX VARIABLES
March 1962, 464 pp., \$7.25 tent.

PRELIMINARY ANNOUNCEMENT OF MEETING

FIVE HUNDRED EIGHTY-EIGHTH MEETING

Yeshiva University
New York, New York
February 22, 1961

The five hundred eighty-eighth meeting of the American Mathematical Society will be held on Thursday, February 22, 1961, at Yeshiva University.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be an address at 2:00 P.M. on "The role of differentiability in the theory of transformation groups" by Professor Richard S. Palais of Brandeis University.

There will be sessions for contributed papers both morning and afternoon. Contributed papers in any area of mathematics will receive the usual consideration, but papers in the fields of

are specially solicited.

The place of the meeting is the Albert Einstein College of Medicine of Yeshiva University, located at Eastchester Road and Morris Park Avenue in the Bronx. It is possible that there will be a change of location to a new classroom and office building which is more centrally located. If so, this will be noted not only in the program of the meeting in these NOTICES for February but also in an announcement mailed directly to institutional and corporate members.

Everett Pitcher
Associate Secretary

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ACTIVITIES OF OTHER ASSOCIATIONS

THE MATHEMATICAL ASSOCIATION OF AMERICA

The Mathematical Association of America will hold its forty-fifth annual meeting from Wednesday to Friday, January 24-26. The program follows.

FIRST MAA SESSION, WEDNESDAY - 2:30 P.M.

Roof garden

LOGIC

- | | |
|-------------|--|
| 2:30 - 3:20 | Some Recent Developments in Logic
Professor L. A. Henkin, University of California, Berkeley and
the Institute for Advanced Study, Princeton |
| 3:30 - 4:20 | Logic in College Curricula
Professor Hartley Rogers, Jr., Massachusetts Institute of Technology |
| 4:30 - 5:20 | Logic in School Curricula
Professor F. B. Fitch, Yale University |

SECOND MAA SESSION, THURSDAY - 9:00 A.M.

Roof Garden

REAL ANALYSIS - A FIRST COURSE?

- | | |
|---------------|--|
| 9:00 - 9:25 | A First Course in Analysis (CUPM)
Professor L. W. Cohen, University of Maryland |
| 9:25 - 9:50 | Reformation in Advanced Calculus
Professor N. E. Steenrod, Princeton University |
| 10:00 - 10:30 | Critique, Rebuttal, Discussion from the Floor
Moderator: Professor B. J. Pettis, University of North Carolina |
| 10:35 - 11:15 | Business Meeting of the Association and the Association's First Award
for Distinguished Service to Mathematics. |

THIRD MAA SESSION, FRIDAY - 9:30 A.M.

Roof Garden

GEOMETRY - TWO APPROACHES

- | | |
|---------------|---|
| 9:30 - 10:20 | Geometric Algebra
Professor Ernst Snapper, Indiana University |
| 10:30 - 11:20 | Axiomatics
Professor Marshall Hall, Jr., California Institute of Technology |
| 11:30 - 12:20 | Retiring Presidential Address: The Narrow Mathematician
Professor C. B. Allendoerfer, University of Washington |

FOURTH MAA SESSION, FRIDAY - 2:00 P.M.

Roof Garden

GEOMETRY AND THE COLLEGE AND SECONDARY CURRICULA

- | | |
|-------------|---|
| 2:00 - 2:50 | Linear Algebra for Underclassmen
Professor Arno Jaeger, University of Cincinnati |
| 3:00 - 3:50 | Axiomatic Geometry for Undergraduates
Professor H. S. M. Coxeter, University of Toronto |
| 4:00 - 4:50 | Affine Geometry for the Secondary School Curriculum
Professor Howard Levi, Columbia University |

SOCIEDAD MATEMATICA MEXICANA

The Sociedad Matemática Mexicana will hold its seventh Congress in the cities of Xalapa and Veracruz, México, from February 26 to March 3, 1962. Members of the AMS are welcome to attend the Congress, and those who are also members of the Sociedad Matemática Mexicana are invited to present research papers on any branch of mathematics or a related field: pure or applied mathematics, teaching, philosophy, or history of mathematics, etc. A Proceedings volume will be published after the Congress.

The deadline for contributions is February 15, 1962. Papers should be sent, typed, to General Secretary, Sociedad Matemática Mexicana, Tabuca 5, México 1, D. F.

MARCH 1962 MEETING OF SIAM

A two-day meeting of SIAM will be held at the California Institute of Technology in Pasadena, California on Friday and Saturday, March 23-24, 1962. This meeting will be dedicated to the memory of H. P. Robertson, late Professor of Physics at the California Institute of Technology. Appropriate to Professor Robertson's life-long interests, invited addresses and a symposium on the "Mathematics of Cosmology and Relativity" will take place on Friday.

Sessions for contributed papers will be held on Friday and Saturday afternoons at 3:30. Abstracts for contributed papers should be mailed to: Professor R. E. Gaskell, Department of Mathematics, Oregon State College, Corvallis, Oregon, before February 15, 1961.

Further details about the meeting will be given in the February issue of the NOTICES.

MEMORANDA TO MEMBERS

CHAIRMEN OF DEPARTMENTS

The Annual List of Chairmen of Departments has been compiled. Copies may be obtained free of charge by writing to the Headquarters Offices, 190 Hope Street, Providence 6, Rhode Island.

NEW

JUST PUBLISHED

AN INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS

by Earl A. Coddington, University of California, Los Angeles

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by B. H. Arnold, Oregon State University

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VISITING FOREIGN MATHEMATICIANS

The foreign mathematicians listed here are in addition to those who were listed on page 476 of the November, 1961 issue of these NOTICES.

Name	Home Country	Host Institution	Period of visit
Agmon, S.	Israel	New York University, Institute of Mathematical Sciences	Sept. 1961-Oct. 1961
Beuhلمان, Hans	Switzerland	University of California, Berkeley	1961-1962
Boyarski, B.	Poland	New York University, Institute of Mathematical Sciences	Feb. 1962-Feb. 1963
Brycman, J.	France	New York University, Institute of Mathematical Sciences	Sept. 1961-Aug. 1962
Eckmann, B.	Switzerland	New York University, Institute of Mathematical Sciences	Sept. 1961-Oct. 1961
Galindo, A.	Spain	New York University, Institute of Mathematical Sciences	Sept. 1961-Feb. 1962
Guerin, Roger A.	France	University of California, Berkeley	July 1961-Aug. 1961
Hinderer, Karl E.	West Germany	University of California, Berkeley	1961-1962
Huber, Peter J.	Switzerland	University of California, Berkeley	1961-1962
Kellerer, Hans G.	West Germany	University of California, Berkeley	1961-1962
Kwun, Kyung Whan	Korea	University of Wisconsin	Sept. 1961-June 1962
Lopuszanski, J.	Poland	New York University, Institute of Mathematical Sciences	Sept. 1961-Dec. 1961
Morton, K. W.	United Kingdom	New York University, Institute of Mathematical Sciences	Sept. 1961-Aug. 1964
Netanyahu, E.	Israel	New York University, Institute of Mathematical Sciences	Sept. 1961-Aug. 1962
Neumann, B. H.	England	New York University, Institute of Mathematical Sciences	Sept. 1961-May 1962
Neumann, H.	England	New York University, Institute of Mathematical Sciences	Sept. 1961-May 1962

NEWS ITEMS AND ANNOUNCEMENTS

THE SOCIETY is pleased to announce that the Russian-English Dictionary for the Mathematical Sciences will appear on December 22, 1961. The low list price of \$7.70 was made possible by the large number of pre-publication orders. There is the usual 25% discount to members.

We apologize for the delay in publication, which was caused by several revisions that appeared necessary so long as it was possible to make substantial improvements.

The Dictionary in its final form is quite complete in the Mathematical Sciences. In addition to the word lists, it contains a detailed grammar of the Russian language, and is thus the only reference required for reading scientific works in Russian. Set in very readable type, it is a reference work of fine quality and great usefulness to all Mathematical Scientists who wish to keep abreast of Russian scientific literature.

PERSONAL ITEMS

Dr. J. L. ALPERIN of the Mathematical Institute at Oxford, England has been appointed a research assistant at Princeton University.

Assistant Professor S. ARAKI of Kyusyu University, Fukuoka, Japan has been appointed to an assistant professorship at Osaka City University, Osaka, Japan.

Dr. R. M. BAER of the California Research Corporation, Richmond, California has accepted a position as Mathematician with the Computing Center at the University of California, Berkeley.

Dr. M. L. BALINSKI has been appointed a Visiting Lecturer at Princeton University.

R. F. BARNES, JR. on leave from the University of California, Berkeley has accepted a position as Head of the Research Section, Information Sciences Department at the Itel Corporation, Lexington, Massachusetts.

Professor H. BAUER on leave from the Universität Hamburg, Hamburg, Germany has been appointed to a visiting associate professorship at the University of Washington, Seattle.

A. T. BEYER of Litton Systems Incorporated, Woodland Hills, California, has accepted a position as Computer Programmer with the Packard Bell Computer Corporation, Los Angeles, California.

Dr. J. R. BUCHI on leave from the University of Michigan has been appointed to a visiting professorship at the Johannes Gutenberg University, Mainz, Germany for the academic year 1961-1962.

Assistant Professor J. B. BUTLER, JR. of the University of Arizona has been appointed to an associate professorship at Portland State College.

Dr. B. R. BUZBY of the Electro Metallurgical Company, Niagara Falls, New York, has accepted a position as Senior Scientific Analyst with the Union Carbide Corporation, New York, New York.

Professor H. CHU of Northwestern University has been appointed to an assistant professorship at the University of British Columbia.

Dr. Z. CIESIELSKI on leave from the Adama Mickiewicz University in Poznan,

Poland has been appointed a Visiting Research Associate at the University of Illinois, and an Assistant at the Adama Mickiewicz University.

Assistant Professor F. L. CLEAVER of the University of South Florida has been appointed to an assistant professorship at the University of Kentucky.

Dr. W. H. COCKCROFT of The University, Hants, England has been appointed a G. F. Grant Professor of Pure Mathematics, The University, Hull, England.

Dr. D. B. COLEMAN has been appointed to an assistant professorship at Vanderbilt University.

Dr. H. E. CONNER on leave from the University of Wisconsin has been appointed a temporary staff member at the Lincoln Laboratory, Massachusetts Institute of Technology.

Dr. W. R. COWELL of the Bell Telephone Laboratories, Murray Hill, New Jersey has accepted a position as Associate Mathematician with the Argonne National Laboratory, Argonne, Illinois.

W. S. CURRIE of the Texaco Company, Bellaire, Texas has been appointed a Lecturer at the University of Houston.

Dr. K. H. DANIEL has been appointed to an assistant professorship at Michigan State University.

Dr. L. DEAN of the Pacific Missile Range, Point Mugu, California has accepted a position as Member of the Technical Staff with Hughes Aircraft, Culver City, California.

Dr. K. L. DE BOUVERE of the University of California, Berkeley has accepted a position as Member of the Research Staff at the University of Amsterdam, Netherlands.

Assistant Professor R. S. DE ZUR of San Diego State College has accepted a position as Assistant Research Scientist at the Martin Company, Denver, Colorado.

Assistant Professor G. DI ANTONIO of Duquesne University has been appointed to an assistant professorship at Fresno State College.

Dr. R. S. DINSMORE has accepted a position as Scientific Systems Engineer at International Business Machines Corporation, San Jose, California.

Dr. J. DJURIC has accepted a position as Chief Engineer at the Institute Mihailo Pupin, Belgrade, Yugoslavia. At present he is on leave as an Associate Professor at the University of New Mexico.

Professor A. DVORETSKY on leave from the Hebrew University, Jerusalem, Israel has been appointed to a visiting professorship at Columbia University.

Dr. M. P. EMERSON has been appointed Professor and Head of the Mathematics Department at Kansas State Teachers College.

Associate Professor CARL FAITH on leave from Pennsylvania State University has been appointed to a temporary membership at the Institute for Advanced Study during the academic year 1961-1962.

Dr. H. FURSTENBERG has been appointed to an assistant professorship at the University of Minnesota.

B. A. FUSARO has been appointed to an assistant professorship at the University of Oklahoma.

R. GOLDBERG of International Business Machines Corporation, White Plains, New York has accepted a position as Computer Programmer with the Weather System Center, Manchester, Connecticut.

Professor L. A. GOODMAN has returned to his position as Professor of Statistics and Sociology at the University of Chicago after having spent 1960-1961 as a visiting professor of Mathematical Statistics and Sociology at Columbia University.

Dr. L. GREENBERG on leave from Brown University has returned after a leave at the University of Copenhagen, Copenhagen, Denmark.

Dr. T. N. E. GREVILLE of the Operational Mathematics Office, U. S. Army Quartermaster Corps, Washington, D. C. has accepted a position as Vice-President of the S. A. Miller Company, Washington, D. C.

Dr. B. GRÜNBAUM has been appointed to an assistant professorship at the Hebrew University, Jerusalem, Israel.

G. HANNAUER has accepted a position as Applications Engineer with Electronic Associates, Incorporated, Princeton, New Jersey.

G. C. HEWITT has been appointed to an assistant professorship at Montana State University.

Professor K. ITO on leave from Kyoto University, Japan has been appointed to a visiting professorship at Stanford University.

Professor C. G. JAEGER of Pomona College has been appointed to a professorship at Claremont Men's College.

Assistant Professor W. D. JAMES of Fresno State College has accepted a position as Project Engineer with the Philco Corporation, Palo Alto, California.

Professor J. A. JENKINS on leave from Washington University has been appointed a temporary member at the Institute for Advanced Study.

J. P. JOHNSON has accepted a position as an associate mathematician with Texaco, Incorporated, Bellaire, Texas.

Dr. R. D. JOSEPH of the Ford Motor Company, Newport Beach, California has accepted a position as Principal Mathematician with Astropower, Incorporated, Costa Mesa, California.

H. KAMEL has been appointed to an associate professorship at Howard University.

Dr. E. L. KAPLAN on leave from the University of California, Livermore has been appointed to an associate professorship at Oregon State University.

Dr. S. KAPLAN has been appointed a Temporary Member of the Institute of Mathematical Sciences, New York University.

R. L. KELLEY of the University of Miami has been appointed a Research Assistant at the University of Michigan.

Dr. B. KELLOGG of Combustion Engineering, Incorporated, Windsor, Connecticut has accepted a position as Mathematician with Westinghouse Electric, Pittsburgh, Pennsylvania.

Dr. H. S. KONIJN on leave from the University of Sydney, Sydney, Australia has been appointed to a visiting professorship at Yale University.

Dr. S. R. KRAFT of the University of Maryland has been appointed an Associate Research Scientist at New York University.

Assistant Professor M. LEES of New York University has been appointed to an assistant professorship at California Institute of Technology.

S. LEFSCHETZ, Director of the Center for Differential Equations at RIAS, Baltimore, Maryland has been honored by

election to the Royal Society of London.

A. S. LESLIE has accepted a position as Member of the Technical Staff with Hughes Aircraft Company, Culver City, California.

Dr. P. E. LONG has been appointed to an assistant professorship at Southern Illinois University.

H. H. LOVE, JR. of Burroughs Corporation, Pasadena, California has accepted a position as Senior Mathematician with General Precision Incorporated, Glendale, California.

Associate Professor E. A. MAIER of the Pacific Lutheran College has been appointed to an associate professorship at the University of Oregon.

D. J. MALLORY has been appointed a Research Assistant at Iowa State University.

Dr. K. MENDER has returned to the Illinois Institute of Technology after lecturing in Austria, England, Holland, Italy, Poland, and Switzerland during the second semester, 1960-1961.

Dr. H. MIZUMOTO of the Tokyo Institute of Technology, Tokyo, Japan has been appointed to an assistant professorship at Okayama University, Okayama, Japan.

Miss A. MUZYKA of the Polytechnic Institute of Brooklyn has accepted a position as Senior Engineer with the Raytheon Company, Bedford, Massachusetts.

Dr. H. NEUMANN on leave from Manchester College of Science, Manchester, England has been appointed a Senior Research Scientist at New York University for the academic year 1961-1962.

E. NISHIURA of the University of Illinois has been appointed an Assistant Staff Member at the Laboratory for Applied Sciences, University of Chicago.

Dr. T. ONO has been appointed to an assistant professorship at the University of British Columbia, Canada.

T. W. PAGE has accepted a position as Mathematician with the National Security Agency, Fort George G. Meade, Maryland.

Dr. G. P. PATIL of the University of Michigan has been appointed to an assistant professorship at McGill University, Montreal, Quebec, Canada.

D. M. PETERSON of Convair Corporation, Ft. Worth, Texas has been appointed

ed a Staff Member at the Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico.

Dr. H. E. RICHERT on leave from the University of Göttingen, Germany has been appointed to a visiting professorship at Syracuse University.

Associate Professor E. A. ROBINSO on leave from the University of Wisconsin has been appointed to a visiting professorship at Uppsala University, Uppsala, Sweden.

Dr. H. ROHRL of the University of Munich, Munich, Germany has been appointed to a professorship at the University of Minnesota.

Dr. P. ROY of the University of North Carolina has been appointed a Temporary Member at the Institute for Advanced Study, Princeton, New Jersey.

Dr. J. SACKS has been appointed to an associate professorship at Northwestern University.

Miss J. E. SAMMET of Sylvania Electric Products Incorporated, Needham, Massachusetts has accepted a position as Boston Advanced Programming Manager at International Business Machines Corporation, Boston, Massachusetts.

Assistant Professor E. J. SCHWEPPE of Iowa State University has accepted a position as Mathematician with the Defense Department, Washington, D. C.

M. A. SEELYE has accepted a position as Analyst with the Service Bureau Corporation, New York, New York.

Dr. M. SHIMRAT has been appointed to an assistant professorship at the University of Alberta, Calgary, Alberta, Canada.

Professor J. A. SIDDIQI of Aligarh University, Aligarh, India has been appointed Professor and Head of the Mathematics and Statistics Department at Muslim University, Aligarh, India.

Dr. J. W. SMITH of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of California, Los Angeles.

Dr. R. L. STANLEY has been appointed to an associate professorship at Portland State College.

Professor W. L. STROTHER on leave from Miami University has been appointed to a professorship at the University of Florida.

Professor E. P. STARKE of Rutgers,

The State University has retired with the title Professor Emeritus and has been appointed to a professorship at Bloomfield College.

Dr. B. STUBBLEFIELD has been appointed to an associate professorship at Michigan State University, Oakland, Rochester.

Dr. P. SWERLING of the RAND Corporation, Santa Monica, California has accepted a position as Head of the Aerospace Department at the Conductron Corporation, Ann Arbor, Michigan.

Dr. W. E. THOMPSON has accepted a position as Staff Research Engineer with the General Motors Corporation, Santa Barbara, California.

Assistant Professor H. TODA of Kyoto University, Kyoto, Japan has been appointed a Research Associate at Northwestern University.

A. M. TOPOLNICKI of the Atomic Power Development Associates, Detroit, Michigan has accepted a position as Senior Mathematical Analyst with the Olin Mathieson Corporation, New Haven, Connecticut.

Dr. H. VAN ROSSUM of the University of Amsterdam, Netherlands has been appointed a Visiting Associate Professor at Michigan State University.

Assistant Professor G. R. VERMA of Fordham University has been appointed to an associate professorship at Birla College, Pilani, Rajasthan, India.

E. J. VOUGHT of Pomona College has been appointed to an assistant professorship at California State Polytechnic College.

Assistant Professor F. J. WAGNER of Marquette University has been appointed to an assistant professorship at the University of Cincinnati.

Dr. E. L. WALTER of New Mexico State University has accepted a position as Mathematician with the White Sands Missile Range, Albuquerque, New Mexico.

H. WALUM of the University of Notre Dame has been appointed a Research Assistant at the University of Colorado.

Assistant Professor R. R. WELLAND on leave from Ohio State University has been appointed a Temporary Instructor at the University of Chicago.

C. M. WHITE of the General Electric Company, San Jose, California has accept-

ed a position as Programmer with the Burroughs Corporation, Pasadena, California.

Dr. R. F. WILLIAMS of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Chicago for two years beginning October 1, 1961.

Dr. S. WIRJOSUDIRDJO has been appointed a Lecturer at the Bandung Institute of Technology, Bandung, Indonesia.

Professor C. T. YANG on leave from the University of Pennsylvania has been appointed a Temporary Member at the Institute for Advanced Study.

B. ZANE has been appointed to an assistant professorship at Central Washington State College.

The following promotions are announced:

J. E. ADNEY, JR., Michigan State University, to an associate professorship.

K. J. ARNOLD, Michigan State University, to a professorship.

W. E. BARNES, Washington State University, to an associate professorship.

C. C. BRAUNSCHWEIGER, University of Delaware, to an associate professorship.

H. N. CARTER, University of Tulsa, to Professor and Counselor to Men.

W. CRAIG, University of California, Berkeley, to a professorship.

P. H. DOYLE, Michigan State University, to an associate professorship.

W. H. DURFEE, Mount Holyoke College, to a professorship.

D. GALE, Brown University, to a professorship.

H. W. GOULD, West Virginia University, to an assistant professorship.

A. G. HADDOCK, Arkansas College, to an associate professorship.

R. E. HUGHS, Lehigh University, to an assistant professorship.

G. JOHNSON, JR., William Marsh Rice University, to an associate professorship.

C. T. LONG, Washington State University, to an associate professorship.

J. M. MITCHELL, Pennsylvania State University, to a professorship.

K. R. MOUNT, Northwestern University, to an assistant professorship.

E. J. PELLICCIARO, University of Delaware, to an associate professorship.

O. W. RECHARD, Washington State University, to a professorship.

E. A. ROBINSON, University of Wisconsin, to an associate professorship.

L. SCHOENFELD, Pennsylvania State University, to a professorship.

M. L. STEIN, University of Minnesota, to a professorship.

F. M. STEWART, Brown University, to a professorship.

E. TOLSTED, Pomona College, to a professorship.

H. F. WEINBERGER, University of Minnesota, to a professorship.

J. WERMER, Brown University, to a professorship.

C. T. YANG, University of Pennsylvania, to a professorship.

T. YEN, Michigan State University, to an associate professorship.

The following appointments to instructorships are announced:

Beloit College: C. P. SEGUIN; Brown University: Dr. A. H. CLARK, T. KAMBAYASHI; University of Chicago: Dr. G. K. LEAF; Idaho State College: V. L. ENGSTROM-HEG; University of Illinois: A. L. PERESSINI; University of Maine: R. W. PACKARD; Massachusetts Institute of Technology: Dr. H. C. RUMSEY, JR.; Miami University: M. R. HARRELL; University of Nebraska: R. E. PEINADO, H. D'ALARCAO; Northwestern University: Dr. E. D. DAVIS; Princeton University: Dr. A. J. SCHWARTZ; Stanford University: Dr. J. B. AX, Dr. S. HELLERSTEIN; Valparaiso University: J. R. SORENSON; Victoria College, British Columbia, Canada: B. L. EHLE; Wake Forest College: J. B. LINDER; Wartburg College: L. CLABORN.

Deaths:

Dr. H. AULBACH of Brooklyn, New York died on June 12, 1960 at the age of 39. He had been a member of the Society for 14 years.

Brother G. LEWIS of La Salle College died on September 8, 1960 at the age of 78. He had been a member of the Society for 14 years.

Professor Emeritus J. E. BURNAM of Hardin-Simmons University died on June 27, 1961 at the age of 78. He had been a member of the Society for 38 years.

Dr. O. K. DE FOE of Saint Louis College of Pharmacy and Allied Sciences died on June 29, 1961 at the age of 64. He had been a member of the Society for 35 years.

Professor W. W. DENTON of Pikeville College died on January 22, 1961 at the age of 79. He had been a member of the Society for 47 years.

Miss R. M. PETERS of the University of New Hampshire died on May 12, 1961 at the age of 55. She has been a member of the Society for 26 years.

Professor H. P. ROBERTSON of California Institute of Technology died on August 26, 1961 at the age of 58. He had been a member of the Society for 38 years.

Mrs. G. C. SMITH of Spelman College died on May 6, 1961 at the age of 52. She had been a member of the Society for 13 years.

Mr. L. A. WOLD of the University of Washington died on May 28, 1961 at the age of 28.

ERRATUM

The announcement on page 418 of the October issue of the NOTICES concerning Dr. A. M. WHITE of the University of Santa Clara should read as follows:

Assistant Professor A. M. WHITE of the University of Santa Clara has been appointed a Member of the Mathematics Research Center, U. S. Army, University of Wisconsin for the academic year 1961-1962.

MEMORANDA TO MEMBERS

TRANSLITERATION CHANGE

The following note will appear on the inside front cover of MATHEMATICAL REVIEWS for 1962: "By a recent agreement with the Editorial Board of Zentralblatt für Mathematik, the system of transliteration from the Cyrillic will be made identical for the two journals, Zentralblatt für Mathematik and Mathematical Reviews. The only change affecting Mathematical Reviews is that, beginning with January 1962, the last two letters of the Russian alphabet will be transliterated "ju" and "ja". Three changes are being made in the system previously used by Zentralblatt.

This change is desirable for several reasons. As far as mathematics is concerned, the problem of representation of the Cyrillic original will now be treated in a completely uniform manner in the two reviewing journals in the Western world. Secondly, the return to the transliteration of these two letters which was used in the first six volumes of MR will avoid certain ambiguities; for example, the Cyrillic for the name Matyaš of the author of the article reviewed in No. 4576 in Volume 21, No. 7 of MR (July-August 1960) is Matnaš, which cannot be recovered from the current system of transliteration.

At the present time there is great interest in transliteration among representatives of other sciences. For example, a system tentatively designated as "AAAS Transliteration System" has just been presented to a Subcommittee on Transliteration of the American Standards Association. This system represents a compromise among the four systems, BGN, BSI, LC-ALA and ISO, at present in use respectively by the Board of Geographic Names, the British Standards Institution, the Library of Congress-American Library Association System, and the International Standardization Organization System. (Scientific Information NOTES of the NSF, Volume 3 No. 4, August-September, 1961, Page 1.)

The AAAS Compromise System, which is still tentative, was adopted with the idea in mind that it could be type-written with the minimum use of diacritical

marks. The transliteration system used by Zentralblatt and MR has the different purpose of unambiguous recovery of the original Cyrillic; in view of the large number of symbols inevitably employed in mathematical printing, the use of diacritical marks is not a serious difficulty.

THE MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The following item is repeated from the November 1961 issue of the NOTICES and gives a more detailed time schedule.

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Annual Meeting at the Sheraton-Gibson Hotel in Cincinnati, Ohio, on January 23, 24, and 25, 1962. The Register will be conducted in Parlors O, P, and Q from 9:00 A.M. to 5:00 P.M. on each of these three days.

There is no charge for registering either to job applicants or to employers, except when the late registration fee for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of \$1 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms and for position description forms. These forms must be completed and returned to Providence not later than January 3, 1962, in order to be included free of charge in the listings at the Annual Meeting in Cincinnati. Forms which arrive after this closing date, but before January 15, will be included in the Register at the meeting for a late registration fee of \$3.00, and will also be included in the printed listings, but not until ten days after the meeting. The printed listings will be available for distribution both during and after

the meeting.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

THE NATIONAL REGISTER OF SCIENTIFIC AND TECHNICAL PERSONNEL

The Mathematical Sciences Section of the Register will maintain a desk during the Annual Meeting at the Sheraton-Gibson Hotel in Cincinnati, Ohio, on January 23, 24, and 25, 1962. The National Register desk will be located in Parlors O, P, and Q. The attendants will be pleased to assist with registrations and to supply information. The National Register as a whole is a responsibility of the National Science Foundation. The Mathematical Sciences Section is operated by the American Mathematical Society with the cooperation of the Association for Computing Machinery, the Association for Symbolic Logic, the Econometric Society, the Industrial Mathematics Society, the Institute of Mathematical Statistics, the Mathematical Association of America, the Operations Research Society of America, the Society for Industrial and Applied Mathematics, and the Society of Actuaries.

SUMMER EMPLOYMENT FOR MATHEMATICIANS AND COLLEGE MATHEMATICS STUDENTS

The Headquarters Office of the Society has compiled a list of opportunities for summer employment for mathematicians and college mathematics students. Members who are interested in summer employment and would like to obtain a copy of this list should write to the Special Projects Department, American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island. There is no charge to members for this list.

Copies will be available at the annual meeting in Parlors O, P, and Q of the Sheraton-Gibson Hotel.

RECIPROCITY AGREEMENT with the MALAYAN MATHEMATICAL SOCIETY

The American Mathematical Society has entered into a reciprocity agreement with the Malayan Mathematical Society, by which members of each may become members of the other by paying half the regular dues. The regular dues of the Malayan Mathematical Society are 10 Malayan dollars a year; therefore an American Mathematical Society member would pay \$1.65. Privileges of membership include free copies of NABLA, the Bulletin of the Malayan Mathematical Society published quarterly.

Those members of the American Mathematical Society wishing to take advantage of this arrangement should write to: The Secretary, Malayan Mathematical Society, Department of Mathematics, The University, Cluny Road, Singapore 10, Malaya.

It is understood that members under the reciprocity agreement spending time in the other country should pay regular dues while they are there.

AFFILIATE MEMBERSHIP IN IRE COMPUTER GROUP

The Professional Group on Electronic Computers, of the Institute of Radio Engineers, wishes to announce to AMS members its affiliate plan, according to which members of professional societies in allied fields are offered an opportunity to affiliate with the Group and to receive a major technical publication in the computer field, the quarterly IRE Professional Group TRANSACTIONS, without the expense of full membership in a second professional society. The PGEC is concerned with the advancement of the electronic computer field and serves to aid in promoting close cooperation and exchange of technical information among its members.

The affiliate membership fee, which includes the TRANSACTIONS, is \$8.50 per year, a rate considerably below the membership fee for the IRE. Application forms may be obtained from The Institute of Radio Engineers, 1 East 79th Street, New York 21, New York, Attention L. G. Cumming, Technical Secretary.

RECIPROCITY AGREEMENT WITH THE REAL SOCIEDAD MATEMATICA ESPANOLA

The American Mathematical Society has entered into a reciprocity agreement with the Real Sociedad Matemática Española by which members of each may become members of the other by paying half the regular dues. The regular dues of members of the Real Sociedad Matemática Española are 150 pesetas plus expenses of sending the publications to foreign members; therefore an American Mathematical Society member would pay \$1.75, which includes the mailing expense of the publications. Privileges of membership include:

- REVISTA MATEMATICA HISPANO-AMERICANA. Appearing six times in three or more issues a year.
- GACETA MATEMATICA. Appearing eight times a year.
- Purchasing the publications of Real Sociedad Matemática Española and those of the Instituto Jorge Juan de Matematicas at reduced rates (discount of 25%).

Those members of the American Mathematical Society wishing to take advantage of this arrangement should write to: Sr. Secretario de la REAL SOCIEDAD MATEMATICA ESPANOLA, Serrano, 123, Madrid 6, Spain.

It is understood that members under the reciprocity agreement spending time in the other country should pay regular dues while they are there.

APPLICATIONS FOR MEMBERSHIP IN GAMM

The Gesellschaft fur Angewandte Mathematik und Mechanik (GAMM) announced that applications for membership under the reciprocity agreement should be addressed in the future to: Professor F. Sommer, Chairman, Membership Committee of GAMM, Klinikstrasse 6, Wurzburg, Germany.

An Invitation to ...

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AND ENGINEERS

from - M. I. T.



The Laboratory's staff of over 1000 under the direction of Dr. C. Stark Draper is engaged in the conception and perfection of completely automatic control systems for the flight and guidance of missiles and space vehicles. Its achievements include the Navy Mark 14 Gunsight, the Air Force A-1 Gunsight, Hermetic Integrating Gyros (HIG), and the Ship Inertial Navigation System (SINS). The Laboratory developed basic theory, components and systems for the Air Force THOR and, later, the TITAN missile. Other accomplishments include the Navy's POLARIS Guidance System.

Recently, the Instrumentation Laboratory was selected by NASA to develop the guidance navigation system for the moon space craft project, APOLLO.

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MEMOIR 39

TOPICS IN FOURIER AND GEOMETRIC ANALYSIS

By Victor L. Shapiro

100 pages; \$1.70 List Price; 25% discount to members.

This memoir consists of three chapters in the Fourier analysis of functions of several real variables, the last chapter dealing also with geometric analysis. Chapter I deals with the expansion of functions defined on the unit sphere and the application of such expansions to the uniqueness theory of functions harmonic in the interior of the unit ball. Chapter II

uses the results of Chapter I to obtain results in the spherical summability of conjugate multiple Fourier-Stieltjes series (conjugacy being taken in the Calderón-Zygmund sense). Chapter III studies differential forms in Euclidean k -space by means of multiple Fourier series. In particular, the concepts of bounded p -variation and $Lip(1,p)$ for $(k-1)$ -forms are defined in a natural manner, and the equivalence of these concepts (well-known for zero forms) is proved.

MEMOIR 40

REGULAR MAPPINGS AND THE SPACE OF HOMEOMORPHISMS ON A 3-MANIFOLD

by Mary-Elizabeth Hamstrom

42 pages; \$1.50 List Price; 25% discount to members.

A proper mapping f of a metric space X onto a metric space Y is said to be homotopy n -regular (h - n -regular) provided that if $y \in Y$, $x \in f^{-1}(y)$ and $\epsilon > 0$, then there is a $\delta > 0$ such that each mapping of a k -sphere, $k \leq n$, into $f^{-1}(y') \cap S(x, \delta)$ is homotopic to 0 on $f^{-1}(y') \cap S(x, \epsilon)$ for arbitrary y' in Y . (The symbol $S(x, \epsilon)$ denotes the set of points in X with distance from x less than ϵ .) The mapping f is completely regular if for each y in Y and $\epsilon > 0$, there is a $\delta > 0$ such that $d(y, y') < \delta$ implies that there is an ϵ -homeomorphism of $f^{-1}(y)$ onto $f^{-1}(y')$. In this Memoir it is proved that if f is an h - n -regular mapping of a metric space X onto a metric

space Y and there is a compact 3-manifold with boundary, M , to which each $f^{-1}(y)$ is homeomorphic and in which each homotopy 3-cell is a 3-cell, then f is completely regular. This result and techniques developed by Eldon Dyer and the author are used to prove that the space of homeomorphisms of a compact 3-manifold with boundary, M , onto itself is LC^n . (A metric space K is LC^n if for each $x \in K$ and $\epsilon > 0$, there is a $\delta > 0$ such that each mapping of a k -sphere, $k \leq n$, into $S(x, \delta)$ is homotopic to 0 in $S(x, \epsilon)$.) Then there are discussed some conditions on X and Y under which (X, f, Y) is a locally trivial fibre space or X is homeomorphic to the direct product $Y \times M$, with f corresponding to the projection mapping.

MEMOIR 41

ISOCLINIC n -PLANES IN EUCLIDEAN $2n$ -SPACE, CLIFFORD PARALLELS IN ELLIPTIC $(2n-1)$ -SPACE, AND THE HURWITZ MATRIX EQUATIONS

by Yung-Chow Wong

112 pages; \$1.80 List Price; 25% discount
to members.

This Memoir is devoted to the study
of the existence and properties of Clifford
parallelism in elliptic spaces of odd dimen-
sion greater than 3. The problem is equi-
valent to one on existence of mutually

isoclinic n -planes in Euclidean $2n$ -space,
which in its turn is closely connected with
the Hurwitz-Radon problem on composition
of quadratic forms. Much information is
obtained through a new and deeper study of
the Hurwitz matrix equations in the field
of real numbers.

COLLOQUIUM PUBLICATIONS

TWO RECENT REPRINTS

Colloquium 24

STRUCTURE OF ALGEBRAS

by A. A. Albert

210 pages; \$6.80 List Price; 25% discount
to members.

Colloquium 25

LATTICE THEORY

by Garrett Birkhoff

283 pages; \$6.30 List Price; 25% discount
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Etwa 550 Seiten, 25 Abbildungen,
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SUPPLEMENTARY PROGRAM - NO. 8

During the interval from October 4, 1961 through November 17, 1961, the papers listed below were accepted by the American Mathematical Society for presentation by title. Readers may wish to refer to page 713 of the November, 1960 issue (No. 49 of these NOTICES) where it is explained in detail that the presentation of papers by title is now dissociated from meetings of the Society. Supplementary program No. 9 will cover the interval from November 18, 1961 through January 9, 1962.

After each title on this program is an identifying number. The abstract of the paper will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these NOTICES.

- (1) Fixed points of continuous mappings into Euclidean n -space
Professor Smbat Abian, University of Pennsylvania, and Professor A.B. Brown, Queens College of the City University of New York (61T-325)
- (2) Fixed points and covering under continuous mappings of a spherical shell
Professor Smbat Abian, University of Pennsylvania, and Professor A.B. Brown, Queens College of the City University of New York (61T-326)
- (3) The homotopy groups of the integral cycle groups
Mr. F. J. Almgren, Jr., Brown University (61T-287)
- (4) Interval functions and the Hellinger integral. Preliminary report
Dr. W. D. L. Appling, Duke University (61T-275)
- (5) Some integral inequalities for uniformly elliptic operators
Professor J. H. Bramble and Professor L. E. Payne University of Maryland (61T-290)
- (6) Concerning Hausdorff transformations and absolutely convergent sequences. Preliminary report
Mr. J. P. Brannen, University of Texas (61T-298)
(Introduced by Professor H. S. Wall)
- (7) Function algebras on an arc
Professor Andrew Browder and Professor John Wermer, Brown University (61T-284)
- (8) Binomial coefficients in an algebraic number field
Dr. L. Carlitz, Duke University (61T-309)
- (9) Summation of some series of Bessel functions
Dr. L. Carlitz, Duke University (61T-310)
- (10) The coefficients in the expansion of certain products
Dr. L. Carlitz, Duke University (61T-311)
- (11) An interpolation problem
Dr. L. Carlitz, Duke University (61T-312)
- (12) Generating functions for powers of certain sequences of integers
Dr. L. Carlitz, Duke University (61T-313)
- (13) Some arithmetic properties of a special sequence of polynomials in the Gaussian field
Dr. L. Carlitz, Duke University (61T-320)
- (14) Some singular quasi-linear Cauchy problems
Professor R. W. Carroll, Rutgers, The State University (61T-295)
- (15) Some analogues of certain arithmetical functions
Professor Eckford Cohen, University of Tennessee (61T-288)
- (16) Isology functors. Preliminary report
Professor A. H. Copeland, Jr., Purdue University and Northwestern University (61T-299)
- (17) Generalized ultrapowers. Preliminary report
Dr. Aubert Daigneault, University of Montreal (61T-327)
- (18) A fixed point theorem with an application to the control of finite state stochastic systems
Mr. J. H. Eaton and Mr. B. H. Whalen, University of California, Berkeley (61T-272)
- (19) On a theorem of D. Ridout in the theory of Diophantine approximations
Professor A. S. Fraenkel, University of Oregon (61T-276)

- (20) Closed extensions of uniformly elliptic, second order differential operators determined by a general class of boundary conditions
Dr. R. S. Freeman, University of California, Lawrence Radiation Laboratory, Livermore (61T-274)
- (21) On mixed problems for the nonlinear heat equation
Professor M. D. George, University of Missouri (61T-286)
- (22) The asymptotic distribution of the number of zero free intervals of a stable process
Professor R. K. Getoor, University of Washington (61T-319)
- (23) On almost-commuting permutations
Professor Daniel Gorenstein, Clark University and Mr. R. I. Sandler, Institute for Defense Analyses, Princeton, New Jersey (61T-321)
- (24) On sums of reciprocals of integers
Mr. Ronald Graham, University of California, Berkeley (61T-282)
- (25) Signature analysis
Dr. M. A. Hyman, IBM Systems Center, Bethesda, Maryland (61T-314)
- (26) On certain stable algebras of degree two
Mr. F. J. Kosier, University of Wisconsin (61T-296)
- (27) On certain nonassociative algebras
Mr. F. J. Kosier, University of Wisconsin (61T-297)
- (28) On a generalization of alternative rings. Preliminary report
Mr. F. J. Kosier, University of Wisconsin (61T-322)
- (29) On 3-manifolds that are not simply connected
Professor K. W. Kwun, Seoul National University, Korea and University of Wisconsin (61T-316)
- (30) Upper semicontinuous decompositions of the n -sphere
Professor K. W. Kwun, Seoul National University, Korea and University of Wisconsin (61T-317)
- (31) A consequence of a theorem of Andrews and Curtis
Professor K. W. Kwun, Seoul National University, Korea and University of Wisconsin (61T-318)
- (32) Primitive modules and division rings
Mr. E. E. Lazerson, Institute for Defense Analyses, Princeton, New Jersey (61T-306)
- (33) The Jacobson radical of a polynomial ring
Mr. E. E. Lazerson, Institute for Defense Analyses, Princeton, New Jersey (61T-307)
- (34) Derivations and nil rings
Mr. E. E. Lazerson, Institute for Defense Analyses, Princeton, New Jersey (61T-308)
- (35) Numerical solutions of the first biharmonic problem by linear programming
Dr. O. L. Mangasarian, Shell Development Company, Emeryville, California (61T-278)
- (36) Remarks on mean value theorems in linear spaces
Dr. W. Mlak, University of Maryland (61T-289)
(Introduced by Professor R. E. Fullerton)
- (37) Differentiable isotopies
Professor Marston Morse, Institute for Advanced Study, Princeton, New Jersey (61T-273)
- (38) On some mutant-preserving transformations
Mr. A. A. Mullin, University of Illinois (61T-281)
- (39) On the mutant cosets of any group
Mr. A. A. Mullin, University of Illinois (61T-305)
- (40) On doubly mutant cosets generated by a skew field
Mr. A. A. Mullin, University of Illinois (61T-323)
- (41) Local homogeneity of Riemannian manifolds
Professor Katsumi Nomizu, Brown University (61T-302)
- (42) Lie algebra of Killing generators
Professor Katsumi Nomizu, Brown University (61T-303)
- (43) Computer searching for orthogonal latin squares of order 10
Dr. E. T. Parker, Remington Rand UNIVAC, St. Paul, Minnesota and University of Minnesota (61T-292)
- (44) A transformation theory for topological space
Professor P. V. Reichelderfer, Ohio State University (61T-291)
- (45) Stellar neighborhoods in polyhedral manifolds

- Professor R. H. Rosen, University of Michigan (61T-277)
- (46) Generalized composition series and the unification of introductory group theory
Dr. Azriel Rosenfeld, Yeshiva University (61T-304)
- (47) On vector-valued functions. Preliminary report
Mr. J. V. Ryff, Stanford University (61T-315)
- (48) The index semigroup of a topological space
Professor A. H. Smith, Long Beach State College (61T-293)
- (49) Minimal regular spaces
Dr. R. H. Sorgenfrey and Dr. M. P. Berri, University of California, Los Angeles (61T-294)
- (50) On Lusin probability spaces
Mr. R. Srinivasan, Wayne State University (61T-324)
(Introduced by Professor A. T. Bharucha-Reid)
- (51) General perturbational solution of the harmonically forced Duffing equation
- tion
Professor R. A. Struble and Mr. S. M. Yionoulis, North Carolina State College (61T-280)
- (52) Spaces of summable functions. Preliminary report
Mr. D. M. Topping, Tulane University (61T-300)
- (53) The intermediate problems, Weyl's first fundamental lemma and the maximum-minimum theory of eigenvalues
Professor Alexander Weinstein, University of Maryland, Institute for Fluid Dynamics and Applied Mathematics (61T-279)
- (54) Trace properties of semigroups of matrices with real quaternion elements
Professor N. A. Wiegmann, George Washington University (61T-283)
- (55) Discrete groups, symmetric spaces, and global holonomy
Dr. J. A. Wolf, Institute for Advanced Study, Princeton, New Jersey (61T-301)

NEWS ITEMS AND ANNOUNCEMENTS

THE NATIONAL ACADEMY OF SCIENCES - NATIONAL RESEARCH COUNCIL announces that applications are now being accepted for Postdoctoral Resident Research Associateships for 1962-1963, tenable only at the U. S. Army Chemical Corps Biological Laboratories, Fort Detrick, Frederick, Maryland. These research associateships provide able and promising young investigators at the postdoctoral level an opportunity for advanced training in basic research in the various branches of the biophysical and biological sciences, including biomathematics, especially biostatistics and biological models.

Awards will be made about April 1, 1962, and tenure, unless otherwise arranged, will begin after July 1, 1962, and continue for one year. The annual gross stipend, subject to income tax, will be \$8,955.

Requests for application forms and additional information should be addressed to the Fellowship Office, National Academy

of Sciences - National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C. Applications, together with their supporting papers, must be received no later than February 1, 1962.

THE SEVENTH CONGRESS ON THEORETICAL AND APPLIED MECHANICS will be held from December 23 to 26, 1961, at the Indian Institute of Technology, Bombay, India, with Dr. D. S. Kothari presiding. The program will consist of an invited address and contributed papers on Elasticity, Plasticity, and Rheology, Fluid Mechanics; Mechanics of Solids; Statistical Mechanics, Thermodynamics and Heat Transfer; Mathematics of Physics and Statistics; Experimental Techniques; and Computational Methods.

Information concerning the Congress may be obtained by writing to Dr. M. K. Jain, Officiating Secretary, The Indian Society of Theoretical and Applied Mechanics, I.I.T., Kharagpur, India.

ABSTRACTS OF CONTRIBUTED PAPERS

The November Meeting in Milwaukee, Wisconsin November 17-18, 1961

584-13. M.-E. HAMSTROM, University of Illinois, Urbana, Illinois. Some global properties of the space of homeomorphisms on a disc with holes.

It has been proved (M.-E. Hamstrom and E. Dyer, Completely regular mappings and the space of homeomorphisms on a 2-manifold, Duke Math. J. vol. 25 (1958) pp. 521-532) that the space of homeomorphisms of a 2-manifold with boundary onto itself that leave the boundary pointwise fixed is locally contractible and that the space of homeomorphisms of an annulus onto itself leaving one of its boundary curves pointwise fixed is contractible and locally contractible. The argument demonstrates that the identity component of the space of homeomorphisms of an annulus onto itself leaving its boundary pointwise fixed is contractible. In the present note it is proved that the identity component of the space of homeomorphisms of a disc with holes onto itself leaving its boundary pointwise fixed is homotopically trivial. This is related, of course, to the well-known theorem of Alexander to the effect that the space of homeomorphisms of an n -cell onto itself leaving its boundary pointwise fixed is contractible and locally contractible as well as to recent results of Fisher, Hamstrom, Sanderson, Kister and McCarty concerning local properties of such spaces. The techniques of the present paper evolve naturally from those of the above cited earlier work. (Received October 4, 1961.)

584-14. R. J. NUNKE, University of Washington, Seattle 5, Washington. Purity.

An exact sequence $0 \rightarrow C \rightarrow E \rightarrow A \rightarrow 0$ of abelian groups is called a pure extension of C by A if C is a pure subgroup of E . The set of pure extensions of C by A forms a subgroup $Z^\omega \text{Ext}(A, C)$ of $\text{Ext}(A, C)$. Here $Z^\omega A$ is the subgroup of elements of infinite height in A . One way to generalize the concept of purity is to consider the composite functors $p^\alpha \text{Ext}$ where p is a prime, α an ordinal number, and $p^\alpha A$ consists of the elements of p -height $\geq \alpha$ in A . An extension in $p^\alpha \text{Ext}(A, C)$ is called p^α -pure. One defines $p^\alpha \text{Ext-projective}$ (-injective) in the obvious way. Each group A has a p^α -pure extension $0 \rightarrow M \rightarrow P \rightarrow A \rightarrow 0$ with P $p^\alpha \text{Ext-projective}$, i.e., $p^\alpha \text{Ext}$ has enough projectives. It also has enough injectives. A $p^\alpha \text{Ext-injective}$ group has the form $D \oplus C$ with D divisible, C cotorsion, and $p^\alpha C = 0$. The following statements are equivalent: (1) If C is cotorsion and $p^\alpha C = 0$; then C is $p^\alpha \text{Ext-injective}$; (2) each group A is a p^α -pure subgroup of a $p^\alpha \text{Ext-injective}$ group such that no $p^\alpha \text{Ext-injective}$ is properly contained between them; (3) an inverse limit of reduced $p^\alpha \text{Ext-injectives}$ is $p^\alpha \text{Ext-injective}$; (4) every subgroup of a $p^\alpha \text{Ext-projective}$ is $p^\alpha \text{Ext-projective}$. These properties are true of all $\alpha < \omega + \omega$ but not of $\omega + \omega$ itself. (Received October 4, 1961.)

584-15. D. D. STANCU, Department of Numerical Analysis, University of Wisconsin, 5520 Sterling Hall, Madison, Wisconsin. Generalizations of a G. G. Lorentz inequality.

In his excellent monograph Bernstein polynomials (Toronto, 1953) Professor G. G. Lorentz proves the following theorem: if the function $f(x)$ has on the interval $[0, 1]$ a continuous derivative $f'(x)$, then the following inequality $|f(x) - B(f; x)| \leq (3/4)\omega_1(1/m^{1/2})$ holds, where $\omega_\nu(\delta)$ is the modulus of continuity of $f^{(\nu)}(x)$ and $B(f; x)$ is Bernstein's polynomial of degree m of this function. The author obtained the following generalization of this theorem: if $f^{(r+1)}(x)$ ($0 \leq r \leq m$) is continuous on $[0, 1]$, then the following inequality: $|f^{(r)}(x) - B^{(r)}(f; x)| \leq (3/4)(1/m^{1/2})\omega_{r+1}(1/m^{1/2}) + rO[(1/m^{1/2})\omega_{r+1}(1/m^{1/2})] + rO(1/m^{1/2})$ holds. For $r = 0$ follows the inequality of G. G. Lorentz. In the case of two variables let $f(x, y)$ be a function defined on the square $D: 0 \leq x, y \leq 1$ and $B(x, y)$ be Bernstein's polynomial of degree (m, n) which corresponds to this function. One obtains the inequality: $|f_{x^r y^s}^{(r+s)}(x, y) - B_{x^r y^s}^{(r+s)}(x, y)| \leq A_{m,n}^{r,s}(1/m^{1/2})\omega_{r+1,s}(1/m^{1/2}, 1/n^{1/2}) + C_{m,n}^{r,s}(1/n^{1/2})\omega_{r,s+1}(1/m^{1/2}, 1/n^{1/2}) + D_{m,n}^{n,s}M_{r,s}(f)$, where A, C and D tend to $3/4, 3/4, 0$ respectively, when $m, n \rightarrow \infty$, $\omega_{r,s}(\delta_1, \delta_2)$ represents the modulus of continuity of $f_{x^r y^s}^{(r+s)}(x, y)$ and $M_{r,s}(f)$ is the supremum of the absolute value of this derivative. For $r = s = 0$ there results the following inequality which corresponds to the inequality of B. B. Lorentz: $|f(x, y) - B(x, y)| \leq (1/m^{1/2})\omega_{1,0}(1/m^{1/2}, 1/n^{1/2}) + (1/n^{1/2})\omega_{0,1}(1/m^{1/2}, 1/n^{1/2})$. (Received October 4, 1961.)

584-16. P. C. HAMMER, 5534 Sterling Hall, Numerical Analysis Department, University of Wisconsin, Madison 6, Wisconsin, and T. J. SMITH, Kalamazoo College, Kalamazoo, Michigan. Conditions implying central symmetry of a convex curve.

Let C be a convex planar body, and let γ be its boundary curve. A diametral chord of C is a chord of maximal length in a family of parallel chords. The following theorem is proved. Theorem. (1) If every chord of C which divides γ in two equal length arcs is a diametral chord, then C has a point of symmetry through which these chords pass. (2) If every chord of C which divides C into two equal areas is a diametral chord, then C has a point of symmetry through which these chords pass. (3) If every chord of C which divides C into equal areas also divides γ into equal length arcs, then C has a point of symmetry through which these chords pass. (Received October 5, 1961.)

The November Meeting in Gainesville, Florida November 17-18, 1961

585-27. J. J. ANDREWS, Florida State University, Tallahassee, Florida. An example of a non-manifold whose product with the line is 4-space.

R. H. Bing in Point-like decomposition of E^3 gives an example of a decomposition of E^3 such that the decomposition space X is not E^3 . Theorem. $X \times E^1 = E^4$. (Received October 4, 1961.)

585-28. D. R. BROWN, Louisiana State University, Baton Rouge 3, Louisiana. Topological semigroups in inverse limit spaces.

Let $\{A_i\}$ be a collection of topological semigroups indexed by a directed set I . Let f_{ij} be a continuous homomorphism of A_i onto A_j . Clearly the inverse limit of the A_i under the f_{ij} is again a topological semigroup. It is shown that if each A_i is compact and the bonding maps are onto, then idempotents and closed ideals are recovered in the limit space. It is also shown that, subject to certain restrictions, the inverse limit of a system of quotient semigroups A_i modulo B_i is topologically isomorphic to the inverse limit of the A_i modulo the inverse limit of the B_i . (Received October 4, 1961.)

ERRATA - Volume 8

B. E. RHOADES, Total relative strength of a Hausdorff method, page 346, Abstract 582-36.

Line 5. For "is true" read "is not true".

Line 6. For "is totally monotone" read "is not totally monotone".

Line 7. For "is non-negative" read "is not non-negative."

Line 9. The line should read "mass function and to show that it is a decreasing function over a portion of the unit interval."

ISRAEL HALPERIN, Sequences of elements in a product space, Page 422, Item (24).

Replace "a product space" by "an inner product space".

Page 441, Abstract 61T-223. Same correction.

ISRAEL HALPERIN, Unitary dilations for commuting contractions, Page 440, Abstract 61T-222.

Line 4. Replace " $(U(n)xy)$ " by " $(U(n)x|y)$ ".

Line 5. Replace " $(T(n)xy)$ " by " $(T(n)x|y)$ ".

J. E. ALLEN, A generalization of convexity, Page 344, Abstract 582-29.

Line 5. For " $X \equiv X^{n-k} \subset L \setminus A^k$ " read " $X \equiv X^{n-k} \subset S \setminus A^k$ ".

The November Meeting in Goleta, California

November 18, 1961

586-22. GILBERT STEINER, Campbell Hall, University of California, Berkeley 4, California.

Transmission problem.

Let Ω_1 and Ω_2 be domains in Euclidean n -space having a portion γ of their boundaries in common. Let $\mathcal{D}$ be a formally self-adjoint, second order, elliptic differential operator on $\Omega_1 \cup \Omega_2$. We take Dirichlet, Neumann, or mixed boundary conditions on $\partial\Omega_i \sim \gamma$, $i = 1, 2$. On γ , we take, $A^i(x)\tilde{u} + A^i(x)\partial\tilde{u}_1/\partial\mu + A^i(x)\tilde{u} + A^i(x)\partial\tilde{u}_2/\partial\mu = 0$, $i = 1, 2$, where $\partial/\partial\mu$ is the conormal derivative, $u_i = u|_{\Omega_i}$, and $\tilde{u}$ is the boundary value of u . With a proper choice of ρ we find $\mathcal{D}$ to be an operator in $L^2(\rho\Omega)$ whose resolvent is a compact operator and whose spectrum lies inside a parabola. (Received October 16, 1961.)

586-23. T. K. BOEHME, University of Washington, Seattle 5, Washington. Transformations

which commute with convolution.

Take p such that $1 \leq p \leq \infty$. Let $\mathcal{L}^p$ be the vector space of complex valued functions on $[0, \infty]$ which are in $L^p[0, N]$ for every $N > 0$. Let juxtaposition denote convolution; thus $k = fg$ is such that $k(t) = \int_0^t f(t-u)g(u)du$ for $t > 0$. It is well known that if f is in $\mathcal{L}^1$ and g is in $\mathcal{L}^p$ then fg is in $\mathcal{L}^p$. The following theorem is proved. Theorem. Let T be a transformation whose domain is $\mathcal{L}^1$ and whose range is contained in $\mathcal{L}^p$, $1 < p \leq \infty$. If $T(fg) = fT(g)$ for all f, g in $\mathcal{L}^1$ then T is a continuous linear transformation of $\mathcal{L}^1$ into $\mathcal{L}^p$ and there is an f_T in $\mathcal{L}^p$ such that $T(g) = f_T g$ for every g in $\mathcal{L}^1$. The identity transformation shows that the theorem cannot be extended to the case $p = 1$; however, the transformations on $\mathcal{L}^1$ to $\mathcal{L}^1$ have already been characterized by the author (Abstract 578-56, Notices Amer. Math. Soc. vol. 8 (1961) p. 239). (Received October 4, 1961.)

586-24. C. B. BELL, San Diego State College, San Diego, California. On the structure of

Markov chains.

A. Rényi (1960) asks: What are the structure theorems for Markov processes comparable to those for independent processes? (See C. B. Bell, 1958, 1961.) Let $(Z, \mathcal{A}, P)$ be an arbitrary probability space; $\{\mathcal{M}_n\}$, a sequence of purely atomic σ -subalgebras of $\mathcal{A}$; $\{\mathcal{X}_n\}$ a sequence of countable state random variables on Z ; P_0 an arbitrary initial vector; and P_n a transition matrix for $n = 1, 2, \dots$. It is found that as in the case of independence there exist elementary relations between the set and measure properties of the process. Lemma. If $\{\mathcal{X}_n\}$ is a Markov chain ergodic of order 1, i.e., with positive $P_0, P_1, P_2, \dots$, then the σ -algebras $\{\mathcal{X}_n^{-1}(\mathcal{O})\}$ are (finitely) set independent. Theorem. The following three conditions are equivalent: (i) the $\{\mathcal{M}_n\}$ are set independent; (ii) for each given $P_0, P_1, P_2, \dots$, there exists a Markov chain $\{\mathcal{X}_n\}$ with P_0 as its initial vector; with transition matrices $\{P_n\}$; and $\mathcal{X}_n^{-1}(\mathcal{O}) = \mathcal{M}_n$, $n = 1, 2, \dots$; (iii) there exists a Markov chain $\{\mathcal{X}_n\}$ ergodic of order 1 and with $\mathcal{X}_n^{-1}(\mathcal{O}) = \mathcal{M}_n$, $n = 1, 2, \dots$. Open problem. What are the extensions of these results to arbitrary Markov processes of orders 1, 2, 3, ... ? (Received October 4, 1961.)

586-25. VICTOR KLEE, University of Washington, Seattle 5, Washington. A conjecture on weak compactness.

Weak compactness has been characterized in terms of intersection properties of convex sets by Šmulian, Dieudonné, Floyd and Klee, Ptak, and Day. James has proved that if C is the unit cell of a separable Banach space and C is not weakly compact, then there exists a linear functional $f \in E^*$ such that $\|f\| = 1$ but $fx < 1$ for all $x \in C$. With $K_n = \{x \in C: fx \leq 1 - 1/n\}$, it is clear not only that $\bigcap_1^\infty K_i = \emptyset$ but also that for each $m \in [0, 1[$, the set mC meets only finitely many of the sets K_i . The theorem of James and previous results lend support to the following Conjecture: A closed convex subset C of a separable Banach space E is weakly compact if and only if every continuous linear functional on E attains a maximum on C . The conjecture reduces at once to the case of a bounded set. In the present note, further support is supplied by proving the following Theorem. Suppose C is a bounded closed convex subset of a separable Banach space, and C is not weakly compact. Then C contains a decreasing sequence K_α of nonempty closed convex sets such that for each $x \in C$ and each $m \in [0, 1[$, the set $x + m(C - x)$ meets only finitely many of the sets K_i . The proof employs the relative extreme points introduced recently by the author. (The paper will appear in Proc. Amer. Math. Soc.)
(Received October 5, 1961.)

THE SIXTY-EIGHTH ANNUAL MEETING IN CINCINNATI, OHIO

January 22-25, 1962

587-1. M. Z. v. KRZYWOBLOCKI, 1 Olds Hall, Michigan State University, East Lansing, Michigan. On a technique of proving ergodic theorem in particular cases.

Birkhoff has established solid fundamentals of the theory of ergodicity by proving the fundamental theorem which has a few formulations, one of them being: with a finite $m(\Omega)$ the ergodicity is equivalent to the metric transitivity. But this theorem is too general to be used in its original form in various particular cases. The author presents a technique of proving the ergodicity in a certain class of problems in which one has to go to details and where the above theorem can be used after all the particular items like the function space, (Banach space in the present case), measure, etc., are being assumed, established and constructed. The technique refers to proving the ergodic theorem for functions representing solutions of certain partial differential equations. By the proper combination of the independent variables, the functions in question are represented as functions of one independent variable. By introduction of a one parameter Abelian group of one-to-one transformations, of linear functionals, of operations on a product space and of statistical and time averages, one shows that in certain cases there exists a metric transitivity. A brief list of equations to which this technique was applied closes the paper. (Received June 6, 1961.)

587-2. J. S. MacNERNEY, University of North Carolina, Chapel Hill, North Carolina. Integral equations and semi-groups.

Let: $\mathcal{O}$ be a linear ordering ($\leq$) of the set S ; N be a complete normed ring (with unit 1, $|1| = 1$); $\mathcal{O}A$ be the set of functions V from $\mathcal{O}$ to N such that (1') $V(x,y) + V(y,z) = V(x,z)$ for each $\{x,y\}$ and $\{y,z\}$ in $\mathcal{O}$ and (2') $|V| \leq f$ on $\mathcal{O}$ for some numerical function f satisfying (1'); $\mathcal{O}M$ be the set of functions W from $\mathcal{O}$ to N such that (1'') $W(x,y)W(y,z) = W(x,z)$ for each $\{x,y\}$ and $\{y,z\}$ in $\mathcal{O}$ and (2'') $|W - 1| \leq g - 1$ on $\mathcal{O}$ for some numerical g satisfying (1''). Theorem. There is a reversible mapping E from $\mathcal{O}A$ onto $\mathcal{O}M$ such that these are equivalent: (1) $W = E(V)$; (2) V is in $\mathcal{O}A$ and if $\{a,b\}$ is in $\mathcal{O}$ then $W(a,b) = {}_a \prod^b [1 + V]$ (the limit, through refinement of subdivisions, of finitely continued products $\prod [1 + V(t_{p-1}, t_p)]$); (3) W is in $\mathcal{O}M$ and if $\{a,b\}$ is in $\mathcal{O}$ then $V(a,b) = {}_a \sum^b [W - 1]$ (the analogous limit of sums $\sum [W(t_{p-1}, t_p) - 1]$). This theorem leads to an integral equation theory which, if S is the real line, extends earlier results [J. Elisha Mitchell Sci. Soc. vol. 71 (1955) pp. 185-200], complementing the theory developed by T. H. Hildebrandt [Illinois J. Math. vol. 3 (1959) pp. 352-373]. If $\mathcal{O}$ arises canonically from a semi-group operation σ on S ($\{x,z\}$ being in $\mathcal{O}$ only in case $\sigma(x,y) = z$ for some y in S , and $\sigma(\sigma(x,a),b) = \sigma(x,c)$ only in case $\sigma(a,b) = c$), the theory connects additive with multiplicative homomorphisms. (Received August 15, 1961.)

587-3. G. F. CLEMENTS, Syracuse University, Syracuse 10, New York. Entropies and nth widths of several sets of functions. Preliminary report.

Let A_B be the set of real valued functions defined on $[a, b]$, satisfying $f(a) = 0$, $f \in \text{Lip } \alpha$, $0 < \alpha < 1$, $\text{Var}_{[a, b]} f \leq B$, and provided with the uniform metric; let A_1, A_2, A_3 be the metric spaces of functions defined on $[0, 1]$, with the distance between two functions being the Hausdorff distance between their graphs, and satisfying respectively: (1) $f(0) = 0$, $f(1) = 1$, f is monotone; (2) $|f| \leq M$, $\text{Var}_{[0, 1]} f \leq B$; (3) $f(x+), f(x-)$ exist for $x \in [0, 1]$, $|f| \leq M$. If $H_\epsilon(A)$ and $d_n(A)$ are the ϵ -entropy of A and the n th width of A respectively [Kolmogoroff and Tichomirow, Arbeiten zur Informationstheorie, III, Berlin, 1960; Kolmogoroff, Ann. of Math. vol. 37 (1936) pp. 107-110], then: Theorem 1. $H_\epsilon(A_B) \times (1/\epsilon) \log(1/\epsilon)$. Theorem 2. Entropies of A_1, A_2, A_3 are respectively of order $1/\epsilon, 1/\epsilon, (1/\epsilon) \log(1/\epsilon)$ and $d_n(A_i) \times n^{-1}$, $i = 1, 2, 3$. Proving the statements about entropy involves constructing and counting ϵ -covers and ϵ -distinguishable sets. The estimates from below of d_n use the idea that functions close enough to linearly independent functions are themselves linearly independent. (Received August 21, 1961.)

587-4. LEE LORCH, University of Alberta, Edmonton, Alberta, Canada, and D. J. NEWMAN, Yeshiva University, 186th Street and Amsterdam Avenue, New York 33, New York. The Lebesgue constants for regular Taylor and $[F, d_n]$ summation of Fourier series.

The indicated Lebesgue constants are shown here to be unbounded. Thus these methods exhibit the du Bois-Reymond and Lebesgue singularities. For example: If $d_n \geq 0$ and S_n is unbounded, then $L(F, d_n) = (2/\pi^2) \log(U_n^2/S_n) + o(1)$, where $S_n = 2 \sum_1^n d_k (1 + d_k)^{-2}$, $U_n = 1 + 2 \sum_1^n (1 + d_k)^{-1}$, $\alpha = -(2/\pi^2)C + (2/\pi) \int_0^1 t^{-1} \sin t \, dt - (2/\pi) \int_1^\infty t^{-1} \{(2/\pi) - |\sin t|\} dt$, where C is Euler's constant. The method is regular if and only if U_n is unbounded. For $L(T_r, n)$ the result is similar and has been obtained previously by K. Ishiguro using a different method (Proc. Japan Acad. vol. 36 (1960) pp. 470-474). The $[F, d_n]$ means were introduced by A. Jakimovski (Michigan Math. J. vol. 6 (1959) pp. 277-290) and generalize the Euler and Stirling-Karamata-Lototsky means. Two additional results are obtained: (1) If S_n is bounded, then the regular $[F, d_n]$ method is equivalent to convergence for the Fourier series of Lebesgue integrable functions (but not necessarily for all (non-Fourier) series). (2) The Euler means are the only summation methods which are both $[F, d_n]$ and Hausdorff methods. Remark: In (2) the only restriction on d_n is $d_n \neq -1$; everywhere else it is assumed that $d_n \geq 0$. (Received August 29, 1961.)

587-5. PETER WERNER, Mathematics Research Center, University of Wisconsin, Madison Wisconsin. On the boundary value problem of perfect reflection in the theory of electromagnetic wave fields.

Methods developed previously (AMS meeting Milwaukee, November 1961, to appear in Arch. Rational Mech. Anal.) are extended to the following vector problem: Let S be a regular closed surface with the exterior D , $\vec{a}$ a twice continuously differentiable tangential field on S and c a continuous function on S . Find a field $\vec{E}$, continuously differentiable in $D + S$, twice continuously differentiable in D , such that (i) $\Delta \vec{E} + k^2 \vec{E} = 0$ in D ($0 \leq \arg k < \pi$), (ii) $\vec{n} \times \vec{E} = \vec{a}$ on S , (iii) $\text{div } \vec{E} = c$ on S ,

(iv) $\lim_{r \rightarrow \infty} \int_{|x|=r} |\partial/\partial \vec{x} \vec{E} - ik \vec{E}|^2 dS = 0$. If $c = 0$, this problem reduces to the usual boundary value problem for stationary electromagnetic wave fields; here $k^2 = \omega(\omega\epsilon + i\sigma)\mu$ (ω = frequency, ϵ = dielectric constant, μ = permeability, σ = conductivity). Theorem 1. The problem formulated above has a uniquely determined solution $\vec{E}$. $\vec{E}$ can be obtained by solving a system of Fredholm integral equations which is uniquely solvable for every k ($0 \leq \arg k < \pi$). Theorem 2. $\vec{E}$ depends analytically on k (resp. if $c = 0$, on ω, ϵ, μ and σ). Theorem 2 includes (for $\sigma \rightarrow 0$) the principle of limiting absorption. The earlier existence proofs (given by H. Weyl and, for $c = 0$, by C. Müller, W. K. Saunders and A. P. Calderón) are essentially based on the second case of Fredholm's alternative. As a consequence, dependence theorems of a more general character (as Theorem 2) could not be established previously. (With the help of the method of dipole solutions introduced earlier (AMS meeting Chicago, April 1961), the results can be extended to the case of non-homogeneous and even anisotropic media. (Received August 31, 1961.)

587-6. H. G. TUCKER, University of California, Riverside, California An estimate of the compounding distribution of a compound Poisson distribution.

A random variable X is said to have a compound Poisson distribution if there exists a distribution function G such that $G(+0) = 0$ and $P[X = n] = \int_0^\infty e^{-\lambda} (\lambda^n/n!) dG(\lambda)$ for $n = 0, 1, 2, \dots$. Let $\{X_n\}$ denote a sequence of independent random variables with the same compound Poisson distribution compounded by G . Let $\hat{P}_n(n)$ denote the proportion of $X_1, \dots, X_N$ which equal n . For any infinite sequence $\underline{a} = \{a_n\}$, define the transformation $T_{\underline{a}} = \{T_{n,\underline{a}}\}$ as follows. If $\Delta_n(\underline{a}) = |a_{i+j}|_{i=j=0}^n > 0$ and $\Delta_n^{(1)}(\underline{a}) = |a_{i+j+1}|_{i=j=0}^n > 0$ for all n , define $T_{n,\underline{a}} = a_n$ for all n . If, for some m , $\Delta_1(\underline{a}) > 0, \dots, \Delta_m(\underline{a}) > 0, \Delta_1^{(1)}(\underline{a}) > 0, \dots, \Delta_m^{(1)}(\underline{a}) > 0$ and $\Delta_{m+1}(\underline{a}) \leq 0$ or $\Delta_{m+1}^{(1)}(\underline{a}) \leq 0$, then $T_{n,\underline{a}} = a_n$ for $1 \leq n \leq 2m+1$, and, for $n > 2m+1$, $T_{n,\underline{a}}$ is defined such that $\Delta_{m+k}(T_{\underline{a}}) = \Delta_{m+k}^{(1)}(T_{\underline{a}}) = 0$ for all $k \geq 1$. Let $\{m_N\}$ be a sequence of odd integers $\rightarrow \infty$, let $\hat{\mu}_N = \{n! \hat{P}_N(n)/\hat{P}_N(0)\}$, and let $\hat{K}_N(x)$ denote the distribution function with at most $(m_N + 1)/2$ points of increase whose first m_N moments are $T_{n,\hat{\mu}_N}$, $1 \leq n \leq m_N$. Define $\hat{G}_N(\lambda) = \hat{P}_N(0) \int_0^\infty \lambda^\tau d\hat{K}_N(\tau)$. Then $\hat{G}_N(\lambda)$ is shown to be a consistent estimate of $G(\lambda)$ in the sense that $\hat{G}_N(\lambda) \rightarrow G(\lambda)$ a.s. as $N \rightarrow \infty$ at all values of λ at which G is continuous. (Received September 11, 19761.)

587-7. L. L. HELMS, University of Illinois, Urbana, Illinois. A Šilov boundary for subregular functions. Preliminary report.

Let A be a generalized second order differential operator on the Banach space $C(\Omega)$ of real-valued bounded continuous functions on the connected locally compact metric space Ω , let V be a non-empty open subset of Ω with compact closure $\bar{V}$ and nonempty boundary V' , and let $D(A, V)$ denote the set of functions x (restricted to $\bar{V}$) which are in the local domain of A at each point of $\bar{V}$. Let P be the set of functions x subregular on V ; i.e., $P = \{x: x \in D(A, V), Ax \geq 0 \text{ on } V\}$. It is assumed that $D(A, V)$ is dense in $C(\bar{V})$ and that $D(A, V)$ contains the constant functions. Using techniques developed by H. Bauer (Arch. Math., vol. 11 Fasc. 3 (1960)), it is proved that V possesses a Šilov type boundary for P ; i.e., there exists a unique, minimal, nonempty, compact subset of V' on which every element of P attains its maximum value. (Received September 11, 1961.)

587-8. H. C. SEBRING, Operations Analysis Unit, Defense Systems Department, General Electric Company, 208 South Geddes Street, Syracuse, New York. Characteristics of a rotating fluid.

The irrotational criterion and velocity potential of a rotating fluid are determined. The fluid is assumed to rotate about the Z-axis with a speed inversely proportional to its radial distance. The potential and criterion are tested by proving that the fluid velocity vector is the gradient of the velocity potential, the latter being a scalar function. Fundamental to the analyses is the reduction of the fluid velocity vector to its axial components. This is accomplished by the solution of simultaneous equations. (Received September 11, 1961.)

587-9. B. E. RHOADES, Lafayette College, Easton, Pennsylvania. A sufficient condition for total monotonicity.

Let $\mu = \{\mu_n\}$ denote a real positive sequence, $f(t)$ a function of class C^∞ for $t > 0$ and such that $f(k) = \mu_k$, $k = 0, 1, 2, \dots$. Define $g(t) = f(t)/f(t+1)$, $h(t) = (-1)g'(t)/g(t)$. Theorem. If $\lim_{t \rightarrow \infty} g(t) = 1$, $g(t) > 0$ and $h(t)$ is totally monotone for $t > 0$, then μ is a totally monotone sequence. Although the theorem is only a sufficient condition for a sequence to be totally monotone, the constructive procedure of the theorem provides new simple proofs that many known sequences are totally monotone. In addition, the theorem has been used by the author for establishing the total monotonicity of some sequences hitherto not investigated. (Received September 19, 1961.)

587-10. T. S. FRANK, Syracuse University, Syracuse 10, New York. Integration on quasigroups. Preliminary report.

A unilateral inversive (UI) quasigroup Q is a set of elements closed under a single-valued binary operation for which left [right]-sided equations have unique solutions, and with the property that for each x in Q there exists x^* in Q such that $(ax)x^* = a$ [$x^*(xa) = a$] for all elements a in Q . Let B be the set of real-valued bounded functions on a UI-quasigroup. By generalizing the relation of decomposition-equivalence (Zerlegungsgleichheit) introduced by Kirsch [Math. Ann. vol. 124 (1952) pp. 343-363] it is shown that an appropriate module $F \subset B$ and a functional I defined on F can be chosen so that I is (1) linear and homogeneous, (2) monotone, (3) invariant (with respect to decomposition-equivalence) and (4) normed, and that these properties uniquely determine I . A second theorem establishes the nontriviality of I (i.e. I not identically zero) in some particular cases. Necessary and sufficient conditions for integrability of functions are given, along with a class of examples of UI-quasigroups. (Received September 19, 1961.)

587-11. ARNOLD LEBOW, University of Michigan, Ann Arbor, Michigan. On normal dilations and spectral sets. Preliminary report.

Let X be a compact subset of the complex numbers. Von Neumann [Math. Nachr. vol. 4 (1951) pp. 258-281] has defined X to be a spectral set for an operator T on a Hilbert space H if $\|f(T)\| \leq \sup\{|f(z)|: z \in X\}$, for every rational function f . Theorem. There is a normal operator N on

a Hilbert space $K \supset H$ such that: (i) spectrum of N is contained in the boundary of X ; (ii) $PN^{\perp}P = T^{\perp}$, where P is the orthogonal projection of K onto H . Using the result of von Neumann that the unit disk is a spectral set for every contraction, the theorem of Nagy [Acta Sci. Math. Szeged vol. 15 (1953) pp. 87-92] on unitary dilations of contractions follows. (Received October 2, 1961.)

587-12. G. R. SELL, 1700-29 Murfin, Ann Arbor, Michigan. Stability, uniform stability and Lyapunov's second method. Preliminary report.

Assume that the solutions $F(t, x_0, t_0)$ of $(*) x' = f(x, t)$, with $f(0, t) = 0$, exist for all $(x_0, t_0) \in D = \{(x, t): |x| \leq M, T \leq t\}$ and for all $t \geq t_0$, where $x \in E^n$ (Euclidean n -space.) Assume also that they are unique and continuous in all variables. Let Σ be the class of all continuous functions $\sigma(t) \geq 0$, $t \geq T$. For $\sigma \in \Sigma$ define I_{σ} as the class of functions $\psi(|x|, t) \geq 0$ defined on D where $\psi(0, t) = 0$ and ψ is continuous, nondecreasing in $|x|$ and satisfies $\int_0^{\infty} \psi(\sigma(t), t) dt < +\infty$. Following J. L. Massera (Ann. of Math. vol. 64 (1956) pp. 182-206) let $V(x, t)$ be a continuous, positive definite function defined on D . Let V' be the total derivative. Hypothesis A: (1) There is a $\sigma \in \Sigma$ satisfying: If $V'(x, t) \geq 0$ on D , then $|x| \leq \sigma(t)$. (2) There is a $\psi \in I_{\sigma}$ satisfying $V'(x, t) \leq \psi(|x|, t)$ on D . Hypothesis B: There is a $\psi \in I_{\sigma_0}$ satisfying $V'(x, t) \leq \psi(|x|, t)$ on D where $\sigma_0(t) = M$. Then B implies A. Theorem. If V' satisfies either A or B, then the solution $x = 0$ of $(*)$ is stable. If in addition, V has an infinitely small upper bound, then $x = 0$ is uniformly stable. If $\psi = 0$ on D this reduces to the familiar Lyapunov theorem. (Received October 13, 1961.)

587-13. J. C. CANTRELL, University of Tennessee, Knoxville, Tennessee and C. H. EDWARDS, JR., University of Wisconsin, Madison, Wisconsin. Almost locally polyhedral curves in E^n .

A simple closed curve J in E^n is said to be locally polyhedral at the point p if there exists a neighborhood U of p such that $\bar{U} \cap J$ is a subcomplex of some combinatorial triangulation of E^n . Results of Guggenheim and Zeeman to the effect that polyhedral 1-spheres do not link or knot in E^n , $n > 3$, are used to prove Theorem 1. If the simple closed curve J in E^n , $n > 3$, is locally polyhedral except at a single point, then J is tame (can be carried by a space homeomorphism onto the boundary of a standard 2-simplex). If J is a simple closed curve in E^n which is locally polyhedral except at the points of a countable set B , then the order of J is defined to be the least ordinal number α such that the derived set $B^{(\alpha)}$ is empty. Using Theorem 1, the following theorem is proved by transfinite induction on the order of J : Theorem 2. If the simple closed curve J in E^n , $n > 3$, is locally polyhedral except at a countable number of points, then J is tame. Theorem 2 implies that a wild simple closed curve in E^n , $n > 3$, must fail to be locally polyhedral at least at the points of a Cantor set. (Received October 13, 1961.)

587-14. WITHDRAWN.

587-15. ALFRED GRAY, University of California, Los Angeles, California and S. M. SHAH, University of Kansas, Lawrence, Kansas. A note on entire functions and a conjecture of Erdős.

Let $\mu(r)$ denote the maximum term of an entire function $f(z) = \sum_0^\infty a_n z^n$, $\nu(r)$ the rank of this term, and $U = U(f) = \limsup_{r \rightarrow \infty} \mu(r)/M(r)$, $u = u(f) = \liminf_{r \rightarrow \infty} \mu(r)/M(r)$, $L = L(f) = \limsup_{n \rightarrow \infty} |a_n^2/a_{n-1} a_{n+1}|$, $l = l(f) = \liminf_{n \rightarrow \infty} |a_n^2/a_{n-1} a_{n+1}|$. (See S. M. Shah, The behavior of entire functions and a conjecture of Erdős, Amer. Math. Monthly vol. 68 (1961) pp. 419-425, for the notations and Conjecture of Erdős.) Theorem 1. Let a_n be ultimately positive and $L(f) = l(f) = \lambda$ where $1 < \lambda < \infty$. Write $a_{n-1}/a_n = \lambda^n \xi(n)$ and define $\xi(n + \beta) = (1 - \beta) \xi(n) + \beta \xi(n + 1)$ where $0 < \beta < 1$; $\gamma(a) = \sum_{j=-\infty}^\infty (1/\lambda)^{(j^2+j-2ja)/2}$ for $0 \leq a \leq 1$. Then (i) $\{\gamma(a)\}^{-1} = \lim_{n \rightarrow \infty} \mu(\xi(n + \beta) \lambda^{n+a})/M(\xi(n + \beta) \lambda^{n+a})$ for any β , $0 \leq \beta \leq 1$. (ii) $1 > \{\gamma(1/2)\}^{-1} = U(f) > u(f) = \{\gamma(0)\}^{-1} > 0$. (iii) For each η with $\gamma(1/2) \leq \eta \leq \gamma(0)$, η^{-1} is a limiting value of $\{\mu(r)/M(r)\}$. Theorem 2. Suppose $f(z)$ satisfies the condition $\liminf_{r \rightarrow \infty} \log M(r)/(\log r)^2 < (4 \log[(\pi + 2)/(\pi - 2)])^{-1}$. Then $U(f) > u(f)$. (Received October 18, 1961.)

587-16. D. H. TUCKER, University of Utah, Salt Lake City 12, Utah. A note on the Riesz representation theorem.

Suppose S_1 is a linear normed space, S_2 is a linear normed and complete space, C is the space of continuous functions from $[0, 1]$ into S_1 with norm $\|g\|_C = \int_0^1 \|g(x)\|_{S_1} dx$, and $B[S_1, S_2]$ is the space of bounded linear transformations from S_1 into S_2 . Theorem. If T is a bounded linear transformation from C into S_2 , then there exists a function $K(x)$ defined and of bounded variation on $[0, 1]$ with values in $B[S_1, S_2]$ such that for each function f in C , $T[f] = \int_0^1 dK(x) \cdot f(x)$. (Received October 19, 1961.)

587-17. W. MLAK, University of Maryland, College Park, Maryland. Integration of the first order differential inequality with distributions.

The considered distributions are taken over the space of complex valued functions C^∞ with compact supports in R^n . Let $u(t)$ be weakly differentiable in t ($t \geq 0$) distribution valued function. The functions $a_i(t)$ ($i = 1, \dots, n$) and $b(t)$ are real valued functions, continuous for $t \geq 0$. It is shown that the inequalities in the sense of distribution $\partial u / \partial t \leq \sum_{j=1}^n a_j(t) \partial u / \partial x_j + b(t) u$, $u(0) \leq 0$, imply $u(t) < 0$, $t \geq 0$. The Fourier transform is used together with some simple properties of the cone of positive functions. (Received October 19, 1961.)

587-18. G. U. BRAUER, Institute of Technology, University of Minnesota, Minneapolis 14, Minnesota. Convolution of sequences.

A summability method is a linear functional ϕ on a space of sequences. This paper defines several types of convolution of sequences (the convolution or multiplication operation is denoted by $*$), and studies the regular methods ϕ such that $\phi(s * t) = \phi(s)\phi(t)$ for each pair of sequences $s = \{s_n\}$ $t = \{t_n\}$ in the domain of ϕ ; in other words regular homomorphisms are studied from a sequence space to the reals. The types of convolution considered are (a) $s * t = \{s_n t_n\}$, (b) $(s * t) = \{\sum_{k=0}^n (s_k - s_{k-1}) \cdot t_{n-k}\}$ or Cauchy multiplication, (c) $(s * t) = \{\sum_{k=0}^n b_{nk} s_k t_k\}$ where $B = (b_{nk})$ is a non-negative, regular, triangular summation matrix, and (d) $s * t = \{\sum_{n=0}^{\infty} b_{nk} s_k t_{n-k}\}$ where B satisfies all conditions stated in (c) and, in addition, $\lim_{n \rightarrow \infty} b_{n, n-r} = 0$ ($r = 0, 1, \dots$). The set of homomorphisms, Φ , are topologized as follows: $\phi_n \rightarrow \phi$ if and only if $\phi_n(s) \rightarrow \phi(s)$, for each sequence s in the common domain of definition of ϕ_n and ϕ . When the multiplication is (a) or (c), Φ is totally disconnected; when it is (b), Φ contains a continuum; when it is (d), Φ may either be totally disconnected or it may contain a continuum. (Received October 23, 1961.)

587-19. R. A. STRUBLE, P. O. Box 5638, State College Station, Raleigh, North Carolina. Oscillations of a pendulum under parametric excitation.

The motion of a simple pendulum which is excited parametrically by small vertical vibrations of its support is examined. Using a technique developed by the author [Nonlinear differential equations, McGraw-Hill, 1962, Chapter 8], the long-period perturbations are displayed in the form of amplitude and phase modulations for general nonstationary oscillations. Special stationary (subharmonic) oscillations are exhibited and their stability characteristics are determined. Excellent agreement with experimentally determined values of the corresponding bifurcation amplitudes, reported by Skalak and Yarymovych [J. Appl. Mech. vol. 7 (1960) pp. 159-164], is obtained. It is shown that the rest (equilibrium) position becomes unstable as the exciting frequency approaches one-half the natural frequency of the (small) free oscillations of the pendulum. (Received October 23, 1961.)

587-20. C. C. BRAUNSCHWEIGER, University of Delaware, Newark, Delaware. An extension of the nonhomogeneous Farkas theorem.

Using the ordinary dot product notation of vector calculus the nonhomogeneous Farkas theorem can be stated as follows: Theorem 1. Consider the system (1) $x \cdot x_i \geq \alpha_i$ ($1 \leq i \leq n$) where the x, x_i are in Euclidean m -space and the α_i are real. The inequality (2) $x \cdot x_0 \geq \alpha_0$ is satisfied for all x satisfying (1) if and only if there exist non-negative numbers β_i ($1 \leq i \leq n$) such that $x_0 = \sum_{i=1}^n \beta_i x_i$ and $\alpha_0 \leq \sum_{i=1}^n \beta_i \alpha_i$. In this paper Theorem 1 is extended to the case for infinite systems of linear inequalities in an arbitrary real locally convex space E as follows: Let A be any nonempty subset of E , α a real valued function on A with $\alpha_x = \alpha(x)$ for each x in A , R the real numbers under the usual topology, $\mathcal{E} = E \times R$ with the product topology, $\mathcal{A} = \{(x, \alpha_x) : x \in A\}$, $\mathcal{A}^*$ the set of all continuous linear functionals on $\mathcal{E}$ such that for each (x, α) in $\mathcal{E}$, $f(x, \alpha) = x^*(x) - \alpha$ where x^* is a continuous linear functional on E and such that $f(x, \alpha_x) \geq 0$ for each (x, α_x) in $\mathcal{A}$, $\mathcal{A}^+$ the subset of $\mathcal{E}$ of all elements (θ, α) with θ the origin of E and $\alpha \geq 0$, $\mathcal{B} = \{(x, \alpha) : f(x, \alpha) \geq 0 \text{ all } f \text{ in } \mathcal{A}^*\}$, and let $C(\mathcal{A})$ denote the conical hull of $\mathcal{A}$. Theorem 2. If $\mathcal{A}^*$ is nonempty, $\mathcal{B} = C(\mathcal{A}) - \mathcal{A}^+$. (Received October 25, 1961.)

587-21. H. F. WEINBERGER, Institute of Technology, University of Minnesota, Minneapolis 14, Minnesota. A bound for solutions of uniformly elliptic equations.

A bound depending only on the ellipticity constant is established for $\int \tau^p \text{grad } G^2 dx$ where G is the Green's function with singularity at the origin of a uniformly elliptic operator of the form $(a^{ij}_{i,j})$. The integer p must be taken sufficiently large to make the integrals converge in the case of Laplace's equation; i.e., $p \geq n - 1$. The proof depends upon a symmetrization using surface and volume moments instead of area and volume. The above bound leads immediately to a bound of the type given by Moser for the value at an interior point of a solution u of $(a^{ij}_{i,j})_{i,j} = 0$ in terms of its L_2 norm. (Received October 26, 1961.)

587-22. F. G. BRAUER, University of Wisconsin, Madison, Wisconsin. On first order differential equations.

The norm of a solution of a system of ordinary differential equations can be bounded above and below by solutions of two related first order equations. Such comparisons, together with analysis of the first order equations, give easy proofs of results of Wintner (Amer. J. Math. vol. 68 (1946) pp. 451-454), Cooke (Rend. Circ. Mat. Palermo vol. 4 (1955) pp. 301-308), Bihari (Acta Math. Acad. Sci. Hungar. vol. 7 (1956) pp. 81-94), and Langenhop (Proc. Amer. Math. Soc. vol. 11 (1960) pp. 795-799) on global existence and bounds for solutions. The same approach gives estimates for the norm of the difference between two approximate solutions, as shown by Antosiewicz in a forthcoming paper. Other bounds can be obtained which are weaker but easier to calculate than these. (Received October 27, 1961.)

587-23. SEYMOUR GOLDBERG, Box 396, University Park, New Mexico. Closed linear operators and associated continuous linear operators.

Suppose X and Y are normed linear spaces. Let T be a closed linear operator with domain dense in X and range in Y . Let $D(T')_1$ denote the domain of the conjugate space T' with norm defined by $\|y'\|_1 = \|y'\| + \|T'y'\|$. Define T'_1 as the operator T' mapping $D(T')_1$ into X' . It is easy to see that $D(T')_1$ is a Banach space. Theorem. $D(T')_1$ is not only a Banach space, but it is also a conjugate space of a Banach space. Moreover, the operator T'_1 is (with the appropriate identification) the conjugate of a bounded linear operator A . It is then shown that answers to questions concerning the ranges and inverses of T and T' are the same as the answers to the corresponding questions concerning the ranges and inverses of A and A' , e.g., T is an onto map and T' has a bounded inverse if and only if A is an onto map and A' has a bounded inverse. (Received October 27, 1961.)

587-24. ROLPH SCHWARZENBERGER, School of Mathematics, The Institute for Advanced Study, Princeton, New Jersey. Jacobians and symmetric products.

Let C be a curve of genus $g > 1$ and J its Jacobian, both defined over an algebraically closed field. Let $\mu: C \times J \rightarrow C$, $\pi: C \times J \rightarrow J$, be the projections on to the first and second factors. There is a Poincaré divisor R on $C \times J$ and a sequence (T_n) of divisors of degree n on C such that the divisor $R + \mu^{-1}(T_n)$ corresponds to a locally free sheaf L_n of rank one. The coherent sheaves $E_n = \pi_* L_n$ and

$F_n = R^1 \pi_* L_n$ on J have exact sequence and duality properties which allow their Chern classes to be calculated. In particular, F_n defines, by a construction of Grothendieck, a projective variety C_n , which is isomorphic to the n -fold symmetric product of C and when $n > 2g - 2$ is a projective fibre bundle over J . This interpretation implies in particular a theorem of Mattuck (Amer. J. Math. vol. 83 (1961) pp. 189-206). (Received October 23, 1961.)

587-25. D. M. TOPPING, Tulane University, New Orleans 18, Louisiana. On order and commutativity in Jordan operator algebras. Preliminary report.

Let S be a Jordan algebra of self-adjoint operators (with or without the identity operator). Then the uniform closure J of S is the real part of a commutative C^* -algebra if and only if the Jordan product of each pair of positive operators in S is positive. This positivity property passes to J and closure under squaring forces J to be a vector lattice. Spectral analysis in the small via a refinement of an argument due to Misonou [Fukamiya, Misonou and Takeda, Tohoku Math. J. vol. 6 (1954) pp. 89-93] shows J to be commutative. Now let A be a C^* -algebra (with or without identity) and let S be the Jordan algebra of self-adjoint operators in A . Then A is commutative if and only if the Jordan product of each pair of positive operators in S is positive. (Received October 30, 1961.)

587-26. M. P. BERRI, University of California, Los Angeles 24, California. Some properties of minimal topological spaces.

As an extension of results in a previous paper, the author investigates various types of minimal topological spaces. Theorem 1. Any minimal Hausdorff subspace of a Hausdorff space is closed. The converse of this theorem, namely, any closed subspace of a minimal Hausdorff space is minimal Hausdorff, is not true and a counterexample is given. A method is given for constructing on a set a Hausdorff topology which is strictly weaker than a given nonminimal Hausdorff topology. Theorem 2. Any minimal regular subspace of a regular space is closed. A counterexample is also given to the converse of this theorem. A method is also given for constructing on a set a regular topology strictly weaker than a given nonminimal regular topology. Theorem 3. Minimal completely regular spaces, minimal normal spaces, and minimal locally compact spaces are all compact. (Received October 30, 1961.)

587-27. A. H. KRUSE, Research Center, Box 756, New Mexico State University, University Park, New Mexico. Some observations on the axiom of choice.

Assume set theory axiomatized without the axiom of foundation, without the axiom of choice (which is the one for sets rather than classes in this paper), and with Urelemente not ruled out. Let (A) be the conjunction of the axiom of foundation and the axiom that there is no Urelement. Let $\mathcal{O}$ be the class of all ordinal numbers, $\mathcal{U}$ the universal class, and $\mathcal{P}(\mathcal{A})$ the power class of $\mathcal{A}$ for each class $\mathcal{A}$. For each power $\mathcal{M}$ and each $\alpha \in \mathcal{O}$, let $H(\mathcal{M}; \alpha)$ be the statement (roughly that there is no α -sequence of powers between $\mathcal{M}$ and $2^{\mathcal{M}}$) indicated in the writer's paper Some developments in the theory of numerations, Trans. Amer. Math. Soc. vol. 97 (1960) pp. 523-553. Theorem 1, below, generalizes the theorem of Tarski and Lindenbaum that the generalized continuum hypothesis implies

the axiom of choice. The proofs of the first two theorems below use results from op. cit. Theorem 1. If $\alpha \in \mathcal{O}$ and $H(\mathcal{M}; \alpha)$ for each power $\mathcal{M} > |\aleph_0|$, then the axiom of choice holds. Theorem 2. If (A) and the axiom of choice hold, then $\mathcal{U}$ is equipotent with a partition of $\mathcal{P}(\mathcal{O})$. Theorem 3. If (A) holds, $\alpha \in \mathcal{O}$, and $\{\mu \in \mathcal{O} \mid H(|\mu|; \alpha)\}$ is too big to be a set, then the axiom of choice holds. (Received October 30, 1961.)

587-28. R. G. LAATSCH, Oklahoma State University, Stillwater, Oklahoma. Constructions of subadditive functions.

A real function f is subadditive on a set H of real numbers if $x, y, x + y \in H$ imply $f(x + y) \leq f(x) + f(y)$. (1) If f is subadditive on the real line, the graph of f is the graph of a subadditive function for a rotation of axes through angle v whenever the graph is that of a single-valued function for every rotation u between 0 and v . (2) If f is a nondecreasing subadditive function on the non-negative integers and g is a nondecreasing concave function on the unit interval with $g(0) = 0$ and $g(1) = 1$, then the function F , defined by $F(x) = f(\underline{x}) + (f(\underline{x} + 1) - f(\underline{x}))g(x - \underline{x})$, where $\underline{x}$ is the unique integer $x - 1 < \underline{x} \leq x$, is subadditive on the non-negative real line E . The theorem may fail for any removal of a condition on f or g . (3) A non-negative concave g on $[0, 1]$ may be extended to E as a subadditive function by $F(x) = g(x - \underline{x})$. (4) The pathological Cantor function (Munroe, Measure and integration, p. 193) constructed with reference to the ternary set is subadditive on the unit interval and an extremal element of the cone of all nondecreasing subadditive functions there. (Received October 30, 1961.)

587-29. J. J. PRICE, Cornell University, Ithaca, New York. A property of orthogonal polynomials.

Let μ be an absolutely continuous measure on the interval $[a, b]$ with density $\rho(x) \geq 0$, and let $\{p_k(x)\}_0^\infty$ be the associated orthonormal polynomials. By a theorem of G. Freud, if (1): $\rho(x) \geq \rho_0 > 0$, then (2): $\sum_{k=0}^n p_k^2(x) = O(n)$ uniformly in any closed interior subinterval. G. Alexits, in his book, Konvergenzprobleme der Orthogonalreihen, raises the question whether (2) holds without (1). It is shown that in general the answer is negative. A large class of measures is constructed for which both (1) and (2) fail. (Received October 30, 1961.)

587-30. T-S. WU, Tulane University, New Orleans 18, Louisiana. A note on almost periodic functions. Preliminary report.

Suppose G is a topological group and let $C_0[G]$ be the ring of all bounded real-valued continuous functions on G . Let $f \in C_0[G]$; we say that f is a right almost periodic function (R.A.P) in Bohr's sense if given $\epsilon > 0$, then $G = KA$ where K is compact such that $|f(ta) - f(t)| < \epsilon$ for $a \in A, t \in G$. Theorem 1. If f is R.A.P. in Bohr's sense, then $\{f^a \mid f^a(t) = f(ta) \text{ for } a, t \in G\}$ is relatively compact in $C_0[G]$ where $C_0[G]$ is topologized by the point-open topology. Theorem 2. If f is R.A.P. in Bohr sense, then f is almost periodic in von Neumann's sense if and only if f is uniformly continuous with respect to the right uniformity of G . Theorem 3. If G is a k -space (cf. D. Gale, Compact sets of functions and function rings, Proc. Amer. Math. Soc. vol. 1 (1950) pp. 303-308), and f is R.A.P. in Bohr's sense,

then f is A. P. in von Neumann's sense (in particular, this is true when G is locally compact or first countable.) (Received November 1, 1961.)

587-31. R. J. BEAN, University of Maryland, College Park, Maryland. On subsets of a disk which contain all the wild points of the disk.

Professor R. H. Bing has asked the following question. Suppose D is a disk in E^3 and α is an arc in $\text{Int } D$. Is there a subdisk D' of D such that $\alpha \subset \text{Int } D'$ and D' lies on a 2-sphere? This question is shown to have a negative answer. Theorem 1. Let D be a disk in E^3 and let X be a cellular subset of D , $X \subset \text{Int } D$, such that X contains all the wild points of D and let U be any neighborhood of X in D . Then there exists a subdisk, D' , of D in U and a homeomorphism, h , taking E^3 onto itself such that $h(D') = D$ and h is the identity on some neighborhood of X in D . Corollary. If α is an arc in $\text{Int } D$ containing all the wild points of D , then there exists a neighborhood of α on D lying on a 2-sphere if and only if D lies on a 2-sphere. Professor R. H. Bing has given an example of a disk which lies on no 2-sphere, yet the wild points are contained on an arc in $\text{Int } D$. Theorem 2. If D is a disk in E^3 such that D has only one wild point, p , with $p \in \text{Int } D$, then D lies on a 2-sphere. An example is given of a disk in E^3 with only two wild points, both in $\text{Int } D$, such that the disk lies on no 2-sphere. (Received November 3, 1961.)

587-32. J. W. BRACE, University of Maryland, College Park, Maryland. Convergence on filters.

Consider a linear space $G(S, E)$ consisting of functions with an abstract set S as their domain and ranges in a locally convex linear topological space E . A net of functions $\{f_\delta, \delta \in D\}$ in $G(S, E)$ is said to converge to f_0 on a filter $\mathcal{F}$ of subsets of S if the net satisfies condition * (see end of abstract). This leads to a locally convex linear topology on $G(S, E)$ if for every continuous pseudo norm on E and f in $G(S, E)$ there is a set F in $\mathcal{F}$ such that $f(F)$ is bounded with respect to the pseudo norm. The choice of F depends on f and the pseudo norm. An example of the type of results obtained is the following extension of a theorem of Arzelà. Theorem. Let $\{f_\delta, \delta \in D\}$ in $G(S, E)$ be a net of continuous functions converging pointwise to a function f_0 on a topological space S . f_0 is continuous if and only if the net converges to f_0 on every convergent filter of subsets of S . *(Every subnet $\{f_\alpha, \alpha \in A\}$ has the property that for each neighborhood V of θ in E and $a_0 \in A$ there exist an F in $\mathcal{F}$ and a finite set $\{\alpha_1, \dots, \alpha_k\} \subset A$, $\alpha_i > a_0$ for $i = 1, \dots, k$, such that for each s in F , $f_{\alpha_i}(s) - f_0(s)$ is in V for at least one α_i .) (Received November 3, 1961.)

587-33. J. M. MITCHELL, McAllister Hall, The Pennsylvania State University, University Park, Pennsylvania. Properties of harmonic functions of three real variables given by the Bergman-Whittaker operator. I.

Let $X = (x, y, z)$ be a point in three dimensional Euclidean space E_3 and L a closed rectifiable curve, not going through the origin, in the complex ζ -plane. Consider the Bergman-Whittaker operator $(1) (1/2\pi i) \int_L g(v, \zeta) d\zeta/\zeta$, $v = (1/2)(iy + z)\zeta^2 + x\zeta + (1/2)(iy - z)$ and g an analytic function of two complex variables v and ζ . [See S. Bergman, *Ergebnisse der Math. Wissenschafte*, vol. 23, 1960.]

If $g = fp$, where p is a meromorphic function of v with an infinity of poles, and f is an entire function of v and ζ , then (1) represents a multiple-valued harmonic function H in a certain domain of E_3 , which may be extended analytically over E_3 except for an exceptional set of lower dimension. H branches along a denumerable set of circles of increasing radii and all passing through the origin, where it has algebraic singularities of a pole-like type; also H has an essential singularity on the negative (or positive) x -axis and may have poles along an infinite set of parallel lines. If p^{-1} and f are entire functions of v of order ρ on $|\zeta| = 1$, growth properties are obtained for H from known results on the minimum modulus of entire functions. (Received November 3, 1961.)

587-34. D. G. DICKSON, 274 West Engineering, University of Michigan, Ann Arbor, Michigan. Solutions of certain linear homogeneous integral equations. Preliminary report.

Let f of exponential type have conjugate indicator diagram P and Borel transform F . Let $A(R)$ be the set of functions analytic in region R . Define $\mathcal{F}: A(R+P) \rightarrow A(R)$ by $\mathcal{F}[\phi(z)] = (2\pi i)^{-1} \int_{c(z)} F(w) \cdot \phi(z+w)dw$ where $c(z)$ is a suitable curve about P . If $\tau \geq 0$ and $S \subset P$, closed contours $\{\Gamma_n\}$, tending to infinity, are (τ, S) associated with f if for each $\epsilon > 0$, $|f(z) \exp(-\beta z)| > \exp[-(\tau + \epsilon)|z|]$ for $\beta \in S$ and z on Γ_n for all large n . If $\beta \in P$, (τ, β) associated contours exist, $\phi \in A(R+P + \{|z| \leq \tau\})$, and $\mathcal{F}[\phi(z)] = 0$ in R ; then ϕ may be expanded in $R + \beta$ in a locally uniformly convergent series whose terms are linear combinations of the functions $z^h \exp(\alpha z)$ where α is a zero of f of order greater than h . If the contours are (τ, P) associated, the expansion holds in $R + P$. The uniqueness of the coefficients yields results on the representation of a function that is a solution of more than one such equation. (Received November 6, 1961.)

587-35. J. R. DORROH, 1902 A West 33rd, Austin, Texas. The product integral solution of an integral equation. Preliminary report.

Suppose S is an additive Abelian group with a norm $\|\cdot\|$, that S is complete with respect to this norm, and that $[a, b]$ is a number interval. If M is the closure of a bounded domain in S , then $B(M)$ denotes the set of all functions from M into S , $LB(M)$ denotes the set of all functions from $[a, b]$ into $B(M)$; if F is in $LB(M)$ then LF denotes the function G from $[a, b] \times M$ such that $G(x, z) \equiv F(x)z$, and $L'B(M)$ denotes the set of all functions F of $LB(M)$ for which LF is continuous and bounded. If F is in $L'B(M)$, K the closure of a bounded domain which contains $F(x)M$ for all x in $[a, b]$, G in $L'B(K)$, G of bounded variation, c in (a, b) , A an interior element of M , and g a continuous nondecreasing function from $[a, b]$ to a number set such that $\|[G(q) - G(p)]z - [G(q) - G(p)]w\| \leq [g(q) - g(p)]\|z - w\|$, then there exists a number b' in $(c, b]$ such that for each x in $(c, b']$ the product integral $Y(x) = \prod_c^x (I + dG \cdot F)A$ exists, and $Y(x) = A + \int_c^x dG \cdot FY$, where the limits in both integrals are taken over refinements of $[c, x]$. If $E(\delta) = \max\|F(x)z - F(x)w\|$ for all x in $[a, b]$ and all $\|z - w\| \leq \delta$, and $\int_0^\infty E(dI)/E$ does not exist, then Y is the only solution of this integral equation. (Received November 6, 1961.)

587-36. H. G. ELLIS, University of Utah, 201 Mathematics Building, Salt Lake City, Utah.
Solutions of first order differential equations, including singular cases. Preliminary report.

The following theorems are established, subject to the hypothesis that (1) $G(x, w)$ is complex for $0 < x \leq a$, w complex and $|w| \leq d$, (2) $u = \int_{0+}^a G(x, 0)dx$, (3) $|G(x, w) - G(x, 0)| \leq H(x, |w|)$, with $H(x, y)$ nondecreasing for $0 \leq y \leq d$ and x fixed, and (4) $\int_0^a H(x, t|u(x)|)dx \leq K(t) \cdot |u(q) - u(p)|$ for $0 < p < q \leq a$, with $K(t)$ nondecreasing for $0 \leq t < T \leq \infty$. Theorem 1. If for some number $t \geq 0$, $t = 1 + K(t)$, $t|u| \leq d$, $Uf = \int_{0+}^a G(x, f(x))dx$ for every continuous function f on the sect $(0, a]$ such that $|f| \leq t|u|$, and U is continuous under uniform approximation, then for some such function f , $f = Uf$. Theorem 2. If G is continuous and f is a function whose existence is asserted in Theorem 1, then f satisfies $f'(x) = G(x, f(x))$ for $0 < x \leq a$. Theorem 3. If $u > 0$, the inequality of (3) is reversed, the inequality of (4) is reversed for every number $t \geq 0$ for which $\int_{0+}^a H(x, t|u(x)|)dx$ exists, and there is no number $t \geq 0$ such that $t = 1 + K(t)$, then there is no function f such that $f = u + \int_{0+}^a |G(x, f(x)) - G(x, 0)|dx$. These theorems are extended to the equation $f = c + \int_{0+}^a G(x, z[f(x) - c])dx$ for c complex and $|z| \leq 1$; they are related to the theorems of Abstract 580-39 of these Notices, vol. 8 (1961) p. 246. (Received November 17, 1961.)

587-37. A. S. FRAENKEL, University of Oregon, Eugene, Oregon. Distance to nearest integer of powers of rational numbers.

Theorem 1: Let a be algebraic $\neq 0$. Let $P \leq Q$ be any positive integers, $\delta > 0$, $0 \leq \nu < 1$, $c > 1$. Let q_n^* be any integer satisfying (1) $0 < q_n^* < cq_n^\nu$, where $q_n = q_n^* Q^n$. There exists a positive constant a , independent of n , such that $\|aq_n^*(Q/P)^n\| > aq_n^{-(\nu+\delta)}$ for all positive integers n for which $\|aq_n^*(Q/P)^n\| \neq 0$. ($\|x\|$ denotes the distance of x to the nearest integer.) Theorem 2: Let $a_1, \dots, a_r$ be any real numbers, P, Q , any positive integers, $a > 0$, $0 \leq \nu \leq 1$. There is a constant $c > 1$ depending only on a, r , and ν , and an integer q_n^* satisfying (1), such that (2) $\|a_i q_n^*(Q/P)^n\| < aq_n^{-\nu/r}$ ($i = 1, \dots, r$) for all positive integers n , with the provision that $a = Q = 1$ if $\nu = 1$. Theorem 3: Suppose that $Q > 1$ if $\nu < 1$, and let $\eta > 0$. For almost all real vectors $(a_1, \dots, a_r)$, the integers q_n^* in (2) satisfy $q_n^* > q_n^{\nu-\eta}$ for all but a finite number of positive integers n . Theorem 1 is based on a Theorem of Ridout (Mathematika vol. 4 (1957)). For $\nu = 0$, it specializes to a Theorem of Mahler (Mathematika vol. 4 (1957)). The case $r = 1$ of Theorem 2 shows that Theorem 1 is best possible, since Theorem 1 is equivalent to the statement that under the given conditions, $0 < \|aq_n^*(Q/P)^n\| < q_n^{-(\nu+\delta)}$ only finitely often for every $\delta > 0$. (Received November 6, 1961.)

587-38. M.-E. HAMSTROM, 101 Altgeld Hall, University of Illinois, Urbana, Illinois.
A decomposition theorem for E^4 .

Theorem. If π is a fixed hyperplane of E^4 and G is an upper semicontinuous decomposition of E^4 into continua such that (1) each element of G lies in a hyperplane parallel to π , (2) if the element g of G lies in the hyperplane π' parallel to π , then $\pi' - g$ is homeomorphic to the complement in π' of a point and (3) for each hyperplane π' parallel to π the decomposition space associated with the subcollection of G consisting of those elements of G that lie in π' is E^3 , then the decomposition space associated with G is E^4 . This theorem complements the work of Bing and McAuley on decompositions

of E^3 and generalizes a similar theorem of Hamstrom and Dyer concerning E^3 and collections whose elements lie in horizontal planes (Completely regular mappings, Fund. Math. vol. 45 (1957) pp. 103-117, Theorem 8). The methods used for E^3 are extended without difficulty to E^4 . Many examples of Bing demonstrate that condition (3) in the statement of the Theorem cannot be removed. An example is given in this paper to demonstrate that condition (2) is also required. (Received November 6, 1961.)

587-39. C. C. HSIUNG, and H. I. NASSER, Lehigh University, Bethlehem, Pennsylvania. Parallel and central transformations of Riemannian manifolds.

Let M^n, M^{*n} be two compact oriented Riemannian manifolds of dimension n (≥ 2) and class C^3 immersed in a Euclidean space E^{n+m} of dimension $n + m$. By a parallel (respectively central) transformation between M^n, M^{*n} we mean a diffeomorphism $f: M^n \rightarrow M^{*n}$ such that the line joining every pair of corresponding points P, P^* is parallel to a fixed line (respectively passes through a fixed point) in E^{n+m} . The object of this paper is to obtain a condition on the p th ($1 \leq p \leq n$) mean curvatures H_{rp}, H^*_{rp} of M^n, M^{*n} relative to a certain pair of corresponding unit normal vectors e_r, e^*_r at every pair of corresponding points P, P^* under an orientation-preserving parallel (respectively central) transformation f of class C^3 between M^n, M^{*n} such that f be a translation (respectively homotheticity). This problem under various special conditions on m, n and p has been studied by various authors since the pioneer work by H. Hopf and K. Voss (Arch. Math. vol. 3 (1952) pp. 187-192). (Received November 6, 1961.)

587-40. G. G. LORENTZ, Syracuse University, Syracuse 10, New York. Approximation by polynomials with positive coefficients.

Let $P_n(x)$ denote linear combinations with positive coefficients of $x^k(1-x)^m$ with $k + m \leq n$. The author studies the degree of approximation of positive continuous functions on $[0,1]$ by the P_n in the uniform norm. The following result is obtained. There is an absolute constant C so that the modulus of continuity $\omega_f(h)$ of f satisfies $\omega_f(h) \leq \omega(h)$ if and only if there is a P_n with $|f(x) - P_n(x)| \leq C\omega(n^{1/2}x^{-1/2}(1-x)^{-1/2})$. Similar results exist which involve the modulus of continuity of the derivative f' . The proofs depend upon the following counterpart of the inequalities of S. Bernstein and Markov: if $P_n(x) \leq M$ on $[0,1]$, then $|P'_n(x)| \leq CMn^{1/2}x^{-1/2}(1-x)^{-1/2}$ and $|P''_n(x)| \leq CMn$ on $[0,1]$. The main theorem explains the relative slowness of the convergence of the Bernstein polynomials to the generating function. (Received November 6, 1961.)

587-41. S. P. LLOYD, Bell Telephone Laboratories, Murray Hill, New Jersey. On decomposition into independent components.

Let $\mathcal{G}$ be a given sub- σ -field of $\mathcal{F}$ in the atomless probability space $(\Omega, \mathcal{F}, P)$. Define $\mathcal{G}^S = \{\Gamma \in \mathcal{F} : P(\Gamma \cap G) = P(\Gamma)P(G) \text{ for all } G \in \mathcal{G}\}$. We say that decomposition holds if the σ -field generated by $\mathcal{G}$ and $\mathcal{G}^S$ is $\mathcal{F}$. It is shown that (3) $\Rightarrow$ (2) $\Rightarrow$ (1), where (1) decomposition holds (2) there exists a random variable independent of $\mathcal{G}$ whose distribution is continuous (3) the Stone space version of the conditional probability $P(\cdot | \mathcal{G})$ [F. B. Wright, Generalized means, Trans. Amer. Math. Soc. vol. 98 (1961) pp. 187-203] is atomless (as a regular Borel measure) with probability 1. An

example where decomposition fails suggests that (3) is close to necessary for (1). (Received November 6, 1961.)

587-42. C. M. PEARCY, P. O. Box 2180, Houston 1, Texas. A complete set of unitary invariants for operators generating finite W^* -algebras of type I.

Let S be the free multiplicative semi-group on the symbols x and y , and denote words in S by $w(x,y)$. Specht [Zur Theorie der Matrizen, II, Jber. Deutsch. Math. Verein. vol. 50 (1940)] showed that the collection of all traces $\sigma[w(A,A^*)]$, $w(x,y) \in S$ is a complete set of unitary invariants (c.s.u.i.) for any $n \times n$ complex matrix A . The author improves this to Theorem: For any n , there is a subset $S_n \subset S$ containing less than 16^{n^2} words such that the collection of traces $\sigma[w(A,A^*)]$, $w(x,y) \in S_n$ is a c.s.u.i. for any $n \times n$ complex matrix A . Furthermore the same traces furnish a c.s. orthogonal invariant for real $n \times n$ matrices. Next, operators which generate a homogeneous, finite, W^* -algebra of type I are considered, and the following result is obtained. Theorem. If A generates an n -homogeneous algebra, a c.s.u.i. (in the given algebra) for A is furnished by the traces $D[w(A,A^*)]$, $w(x,y) \in S_n$, where $D(\cdot)$ is the unique Dixmier center-valued trace. Finally, operators generating the most general finite W^* -algebra of type I are considered, and a c.s.u.i. for such operators is given. (Received November 6, 1961.)

587-43. L. MOSER and J. R. POUNDER, University of Alberta, Edmonton, Canada. On the square of a polynomial.

Let $f(x)$ be a polynomial of degree n with non-negative coefficients and let $S(x) = f^2(x) = \sum_{i=0}^{2n} s_i x^i$. Several problems in probability theory and in the theory of bases in additive number theory lead to questions of estimating how nearly equal, in various senses, the coefficients of $S(x)$ may be. If $\delta_i = f^2(1)/(2n+1) - s_i$, it is proved that (1) $\sum_{i=0}^{2n} |\delta_i| \geq 0.151 f^2(1)$ and (2) $\sum_{i=0}^{2n} \delta_i^2 \geq .062 f^4(1)/(2n+1)$. The methods of proof involve use of special Fourier series and certain identities for the values of a complex polynomial at regularly spaced points on the unit circle. Special cases which reveal to what extent (1) and (2) might possibly be sharpened are considered. Analogous problems arising out of consideration of continuous rather than discrete probability distributions are also treated. (Received November 6, 1961.)

587-44. DAVID SCHROER, The University of Rochester, Rochester 20, New York. Nested pairs of well-formed occurrences in languages with quantifiers.

(See Abstract 578-51, Notices Amer. Math. Soc. vol. 8 (1961) p. 237.) As is known, any language with formal quantifiers can be naturally imbedded in a canonically associated system of lambda conversion, in which the set of all (generalized) well-formed occurrences (woccs) is generated from singleton occs of variable- and constant-symbols by "structural induction" using the syntactic operations of combination and abstraction (λ -quantification) -- such an imbedding being intermediate to imbedding in a combinatory logic (see Abstracts 548-44 and 557-6, Notices Amer. Math. Soc. vol. 5 (1958) p. 485 and vol. 6 (1959) p. 176). Let NstPr be the relation between woccs A, F such that $\langle A, F \rangle \in \text{NstPr}$ iff F is a part of A -- i.e., iff $F \subseteq A$ and F is not the occ of a variable in a quantifier

occurring in A . Then for each wocc A , the set $\mathcal{L}A = \{F \mid \langle A, F \rangle \in \text{NstPr}\} \cup \{\emptyset\}$ forms a finite lattice with greatest and least elements which, though not always modular, has the Kleene property that for $F, G \in \mathcal{L}A$, if $F \cap G \neq \emptyset$ then $F \subseteq G$ or $G \subseteq F$. Having justified proof and definition by "reverse structural induction", define the characteristic sequence $\chi(A, F)$ to be the expr in the markers $\bar{L}, \bar{R}, \bar{\Lambda}$ which indicates how to reach F going inward from A . It is then easy to treat such concepts as corresponding parts (of two woccs) and to give reasonable statements and proofs of theorems concerning the structure of a wocc before and after syntactic operations based on replacement and substitution. (Received November 6, 1961.)

587-45. J. P. HEMPEL, University of Wisconsin, Madison 6, Wisconsin. A surface in S^3 is tame if it can be deformed into each complementary domain.

By using a characterization of tame surfaces given by R. H. Bing (Abstract 546-47, Notices Amer. Math. Soc. vol. 5 (1958) p. 365), the following theorem is established. Suppose M is a closed 2-manifold in the 3-sphere, S^3 , with the property that for each component U of $S^3 - M$ there is a homotopy $h: M \times [0, 1] \rightarrow \bar{U}$ such that $h(x, 0) = x$ and $h(x, t) \in U$ for $t > 0$. Then M is tame in S^3 . As an application of this theorem the following is proved. If M is a closed 2-manifold which is tame in S^3 and f is a map of S^3 onto itself such that $f|_M$ is a homeomorphism and $f(S^3 - M) = S^3 - f(M)$, then (a) $f(M)$ is tame, and (b) if in addition the closure of each component of $S^3 - M$ is a cube with handles, then $f|_M$ can be extended to a homeomorphism of S^3 onto itself. An example is given to show that the additional hypothesis of (b) is necessary. (Received November 16, 1961.)

587-46. B. RAMACHANDRAN, Catholic University of America, Washington 17, D. C. On the type of an entire characteristic function of finite order.

Elsewhere, the author has obtained necessary and sufficient conditions for a distribution function (d.f.) to have an entire characteristic function (c.f.) of given order (greater than or equal to one). Here, conditions for a d.f. to have an entire c.f. of given finite order greater than one and given type (maximal, intermediate, or minimal) are obtained. Next, a relation between the type of an entire c.f. of order one and of exponential type and the extremities of the corresponding "finite" d.f. is obtained. As a corollary, it is shown that there cannot exist an entire c.f. of order one and of minimal type. (Received November 16, 1961.)

587-47. D. L. KREIDER and R. W. RITCHIE, Dartmouth College, Hanover, New Hampshire. Elementary functionals and definition by recursion.

The frequent use of the recursion theorem in dealing with the recursive functions R suggests that known subclasses of R which are in some sense more constructive than R itself (e.g. the elementary, primitive recursive and multiply recursive functions) be viewed as the closures of restricted classes of functions under the operation of adjoining minimal fixed points of the members of a suitable class of recursive functionals. Starting from a definition of " $\emptyset$ is easier to compute than ψ " (for functions partial recursive in partial functions) based on the amount of storage required in their computation, the notion of A -functional on X is defined for any class A of partial recursive functions and

class X of partial functions as follows: $F: X \rightarrow X$ is an A -functional on X if $(\exists n)(\forall x \in X)[F(x) = \phi_n^x$ and $(\exists \psi \in A)(\phi_n^x$ is easier to compute than $\psi)]$. Let $\hat{A}$ be the class of partial recursive functions obtained from A by adjoining the minimal fixed points of A -functionals on A and closing under composition and explicit transformations. Let $\hat{\mathcal{C}}$ be the class of elementary functions. Theorem. $\hat{\mathcal{C}}$ is the entire class of partial recursive functions. (Received November 17, 1961.)

587-48. D. L. KREIDER and R. W. RITCHIE, Dartmouth College, Hanover, New Hampshire.
Predictably computable functionals.

The predictably computable functionals are recursive functionals which provide a uniform way of predicting the complexity of the output function given an oracle to supply the input function. The definition is a formalization of the following: a Turing machine Z is said to compute ψ predictably in ϕ (for partial functions ϕ and ψ) if there is a non-negative integer k independent of ϕ such that upon receiving an integer j , $0 \leq j \leq k$, (i) Z computes ψ_j in ϕ where ψ_j bounds the amount of storage used by Z in computing ψ_{j-1} , (ii) $\psi_0 = \psi$, and (iii) Z acts as a finite automaton in computing ψ_k in ϕ . A functional F defined on a class X of partial functions is then called predictably computable if there is a Turing machine which computes $F(\psi)$ predictably in ψ for all $\psi \in X$. Even though some elementary functionals on $\hat{\mathcal{C}}$ (see the preceding abstract for definitions) are not predictably computable since the required storage is not uniformly predictable, an extension of the arguments used above yields the strengthened Theorem. Every partial recursive function can be obtained by an explicit transformation from the minimal fixed point of a predictably computable functional. (Received November 17, 1961.)

587-49. E. C. SCHLESINGER, Box 328 Wesleyan Station, Middletown, Connecticut.
On the Wiener - Perron method for the Dirichlet problem. Preliminary report.

On a Riemann surface R let Ω be a region with compact boundary Γ that is regular for the Dirichlet problem and on which the maximum principle is satisfied. If f is a bounded function on Γ we denote by $V(f)$ the family of subharmonic functions v on Ω that satisfy $\limsup v(z) \leq f(\zeta)$, where $z \in \Omega$, $z \rightarrow \zeta$, for every $\zeta \in \Gamma$. Then it is well-known that the function $u_f(z) = \sup\{v(z) | v \in V(f)\}$ is harmonic in Ω and satisfies $(*) \liminf f(\zeta) \leq \liminf u_f(z) \leq \limsup u_f(z) \leq \limsup f(\zeta)$, ($\zeta \in \Gamma$ and $z \in \Omega$ tend to $\zeta^* \in \Gamma$). We define $W(f)$ dually to $V(f)$ and form $\bar{u}_f(z) = \inf\{w(z) | w \in W(f)\}$. $\bar{u}_f$ in place of u_f also satisfies the inequality $(*)$ and, moreover, $u_f \leq \bar{u}_f$. For a fixed point $z \in \Omega$ we consider two classes of bounded functions on Γ : $S_z = \{f | u_f(z) = \bar{u}_f(z)\}$ and $L_z = \{f | f \in L^1\}$, where L^1 is the Daniell-Lebesgue class generated from the continuous functions on Γ by the Daniell extension of the integral $I(f) \equiv u_f(z)$. (Cf. L. H. Loomis, Abstract harmonic analysis, Chapter III.) Without further restrictions on the boundary Γ it is proved that these two classes are equal. It follows that L_z is independent of z , since this is obviously true of S_z . (Received November 6, 1961.)

587-50. CHOY-TAK TAAM, Georgetown University, Washington 7, D. C. A Daniell's approach to integration on locally compact spaces.

Given a locally compact Hausdorff space S and denote by L the vector lattice of all real-valued continuous functions in S with compact support. Let I be a non-negative linear functional on L . A set E in S is called a set of measure zero if there exist an upward directed family $\{\phi_\alpha\}$ of functions in L ,

partially ordered with respect to the relation $\leq$, such that $\phi_a(x) \rightarrow \infty$ for every x in E and $I(\phi_a)$ is bounded. A larger class $\bar{L}$ of functions in S is formed by requiring $f \in \bar{L}$ if there exists an upward directed family $\{\phi_a\}$ of functions in L such that $I(\phi_a)$ is bounded and $\phi_a(x) \rightarrow f(x)$ in S except possibly at a set of measure zero; the integral $I(f)$ of f is defined to be $\lim I(\phi_a)$. Now a still larger class L_1 of functions in S is formed by requiring $f \in L_1$ if $f = f_1 - f_2$ in S , except possibly at a set of measure zero, for some functions f_1, f_2 in $\bar{L}$, and define $I(f) = I(f_1) - I(f_2)$. Every function f in L_1 is called summable and $I(f)$ its integral. Let L_* be the set of all bounded functions in L_1 which vanish off compact sets and let $B(L_*)$ be the smallest monotone family of real-valued functions in S containing L . The functions in $B(L_*)$ are called Borel measurable. The fundamental theorems of integration and measure theory are easily established. (Received November 6, 1961.)

587-51. M. K. FORT, Jr. University of Georgia, Athens, Georgia. Homogeneity of infinite products of manifolds with boundary.

Let $M_1, M_2, M_3, \dots$ be compact manifolds with nonempty boundaries, and let $P = M_1 \times M_2 \times M_3 \times \dots$. It is shown that P is homogeneous with respect to closed and countable subsets. This result generalizes a theorem which V. L. Klee, Jr. has proved for the Hilbert cube. The proof is obtained by using category arguments for the space of all homeomorphisms of P onto itself. It is also shown that if A and B are countable dense subsets of P , then there is a homeomorphism h of P onto P such that $h[A] = B$. (Received November 6, 1961.)

587-52. J. C. WILSON, Florida Presbyterian College, St. Petersburg, Florida, and R. F. RINEHART, Case Institute of Technology, Cleveland, Ohio. The relation between continuity and differentiability of functions on algebras.

Let $\mathcal{A}$ be a finite dimensional associative linear algebra with identity over the real or complex field $\mathcal{F}$, and let f be a function with domain and range in $\mathcal{A}$. A generalized difference quotient definition of differentiability and derivative of f at a point $a \in \mathcal{A}$ has been given for the case where $\mathcal{A}$ is a total matrix algebra [Proc. Amer. Math. Soc. vol. 8 (1957) pp. 329-335]. That definition, and the proofs of uniqueness, and of differentiability and derivatives of sums and products of differentiable functions, given in the referenced paper, are equally valid for any $\mathcal{A}$ of above type. However, the proof of the theorem that if f and g are differentiable at $a \in \mathcal{A}$, then fg is differentiable and $(fg)'(a) = f'(a)g(a) + f(a)g'(a)$, employed the additional assumption that at least one of f, g is continuous at a . It was conjectured that (A) this assumption appears essential since (B) differentiability at a point does not seem to imply continuity at the point. In the present paper this double conjecture is settled by disproving A and proving B. Theorems are included which exhibit the essentialities of the reason for the validity of B, and settle in the negative a conjecture of N. J. Fine that a function differentiable in a neighborhood of a is continuous at a . (Received October 30, 1961.)

587-53. ALBERT NIJENHUIS, Institute for Advanced Study, Princeton, New Jersey and W. B. WOOLF, University of Washington, Seattle, Washington. Some complex integration problems.

Theorem I. In every $2n$ -dimensional manifold with torsion-free almost-complex structure of class $C^{2+\alpha}$ ($0 < \alpha < 1$) there is a compatible complex structure of class $C^{3+\alpha/n}$. Theorem II. In every manifold with almost-complex structure of class $C^{1+\alpha}$ there are local holomorphic curves of class $C^{2+\alpha/n}$ through every point in every complex tangent direction. Proofs follow the pattern of Newlander-Nirenberg [Ann. of Math. vol. 65 (1957) pp. 391-404], with integral operators T and $\bar{T}$ below. Theorem I gives their result under weaker hypotheses. For functions f on $D = \{z \in \mathbb{C} \mid |z| \leq R\}$, with boundary C , let $(Tf)(z) = (-1/2\pi i) \int_D (f(\zeta)/(\zeta - z)) d\bar{\zeta} \wedge d\zeta$ and $(Sf)(z) = (1/2\pi i) \int_C (f(\zeta)/(\zeta - z)) d\zeta$; if f is defined on $D^n \subset \mathbb{C}^n$, let $T^j f$, $S^j f$ be obtained by action of T or S on the j th coordinate; let action on differential forms be defined by action on the components. For $(*) \omega = \sum a_{i_1 \dots i_p j_1 \dots j_q} dz^{i_1} \wedge \dots \wedge dz^{i_p} \wedge d\bar{z}^{j_1} \wedge \dots \wedge d\bar{z}^{j_q}$ of type (p, q) (summation over $i_1 < \dots < i_p$, $j_1 < \dots < j_q$) define $\pi_j \omega$ by $(*)$ with the factor $d\bar{z}^{j_1}$ deleted and summation rule: j fixed, $i_1 < \dots < i_p$, $j = j_1 < \dots < j_q$. Define $T \omega = T^1 \pi_1 \omega + S^1 T^2 \pi_2 \omega + \dots + S^1 \dots S^{n-1} T^n \pi_n \omega$. Theorem III. For ω of class C^1 and type (p, q) with $q \geq 1$, $\omega = d'' T \omega + T d'' \omega$ [cf. Nickerson, Proc. Amer. Math. Soc. vol. 9 (1958) pp. 183-188]. (Received November 8, 1961.)

587-54. N. J. ROTHMAN, University of Rochester, Rochester 20, New York. On the uniqueness of character semigroups.

Let S and T be compact connected commutative semigroups. Let S^* denote the semigroup of all continuous homomorphisms of S into the multiplicative complex disk. This note is concerned with the question, if S^* and T^* are separating families for S and T , and S^* and T^* are isomorphic, when are S and T isomorphic and homomorphic under the same mapping? It is shown that, if S and T are standard threads, then the conclusion is true. The extension of this result to these semigroups S algebraically irreducible between 0 and 1 is shown to be false. However, with the additional hypothesis that, S and T are both algebraically irreducible between 0 and 1 and are such that for any two idempotent elements e and f , with $e < f$ and $H(e)$ and $H(f)$ nontrivial, there is an idempotent element g such that $H(g) = \{g\}$ and $e < g < f$, then the conclusion again follows. Now, each compact connected semigroup with 0 and 1 contains a sub semigroup P algebraically irreducible between 0 and 1, and if P is not a standard thread, there is a compact connected semigroup T with T^* isomorphic to S^* and T not isomorphic to S . It is clear that T and S must be of the same class in order to establish the desired conclusion. Other results will also be mentioned. (Received November 8, 1961.)

587-55. J. C. C. NITSCHKE, University of Minnesota, Minneapolis, Minnesota. The extension of minimal surfaces intersected in convex curves by parallel planes.

A doubly-connected minimal surface S , defined in the slab $|z| < k$, is called of class $C(k)$ if its intersections S_c with every plane $z = c$ ($|c| < k$) are convex curves. It is known that the catenoids are the only minimal surfaces of class $C(\infty)$. (The same is true, more generally, if all curves S_c are starshaped.) The present paper gives a quantitative result: "Given a convex curve S_0 of length L , different from a circle. Let a and b be the minimum value and the maximum value of its curvature,

There exists a constant K , depending only on L and the quotient b/a , with the following property. If S is a minimal surface of class $C(k)$ containing S_0 , then $k \leq K$." (Received November 8, 1961.)

587-56. D. B. A. EPSTEIN, Institute for Advanced Study, Princeton, New Jersey. Steenrod reduced powers in Ext. Preliminary report.

Suppose there is an abelian category, in which a commutative, associative tensor product is defined. Suppose that the tensor product is an exact functor. Let C be a commutative associative coalgebra and A a commutative associative algebra in the category. Then reduced powers in $\text{Ext}^*(C, A)$ can be defined. If $pA = 0$, these are the reduced p th powers, where p is a prime. The p th powers satisfy the Cartan formula and Adem relations but not necessarily $P^0 = 1$. If the category is the category of sheaves annihilated by p on a fixed topological space, one obtains reduced powers in the cohomology of the space with coefficients in a commutative sheaf of rings annihilated by p . (Received November 1, 1961.)

587-57. MARVIN MARCUS, University of British Columbia, Vancouver 8, B. C., Canada, and HENRYK MINC, University of Florida, Gainesville, Florida. Disjoint pairs of sets and incidence matrices.

Let $x_1, \dots, x_n$ be n distinct objects and let $S_1, \dots, S_n$ be n subsets of these objects satisfying the following two conditions: (a) For no s , $s = 1, \dots, n-1$, do s of the sets $S_1, \dots, S_n$ each consist of a subset of some fixed s of the x_j . (b) No two of the sets S_i intersect in precisely one of the x_j . Then the maximum possible number of nonintersecting pairs of sets, S_p and S_q , $1 \leq p < q \leq n$ is: $n(n-4)/2$ if n is even and $n \geq 6$, $(n(n-4)-3)/2$ if n is odd and $n \geq 7$, and 0, 0, 1, 2 if $n = 2, 3, 4, 5$, respectively. (Received November 9, 1961.)

587-58. J. E. MAXFIELD, 205 Walker Hall, University of Florida, Gainesville, Florida. An iterative procedure for finding the zeros of a polynomial.

Given a polynomial $f(x) = x^n + a_{n-1}x^{n-1} + \dots + a_0$ having known zeros, an iterative procedure is developed that converges to the $n+1$ zeros of $g(x) = x^{n+1} + f(x)$. (Received November 9, 1961.)

587-59. J. GILLIS, c/o Institute for Space Studies, 475 Riverside Drive, New York 27, New York, and M. SHIMSHONI, Weizmann Institute of Science, Israel. A numerical integration of the Navier-Stokes equations.

The Navier-Stokes equations have been integrated numerically for the inlet region of a straight circular pipe. Several procedures were first tried unsuccessfully and the reasons for their failure are analyzed. The integration was finally achieved by a relaxation method which is described. No approximations were made other than the finite difference approximation to the differential equations. The numerical solution substantially agrees with both Schiller's calculation and Nikuradse's experimental determination of the length of the region of transition to parabolic flow. (Received November 9, 1961.)

587-60. P. T. CHURCH, Institute for Advanced Study, Princeton, New Jersey and
m. E. MAHOWALD, Syracuse University, Syracuse 10, New York. Measure preserving functions on locally compact spaces.

Let μ and ν be σ -finite Borel measures on locally compact metric spaces M and N , respectively. A function $f: M \rightarrow N$ is measure preserving (not the usual definition) if: (1) for each measurable set Y in N , $f^{-1}(Y)$ is measurable, and (2) whenever X and $f(X)$ are each measurable, then $\mu(X) = \nu(f(X))$. Let $F = \{p \in M: f^{-1}(f(p)) \neq p\}$. Theorem I. A measure preserving function f is one-to-one except on a set whose intersection with each Borel set has measure zero. Theorem II. If M is separable, μ and ν are positive on open sets, and f is a continuous measure preserving function, then F is an F_σ set of the first category, $\mu(F) = 0$, and f is one-to-one on $M-F$. If M is compact, then the restriction of f to $M-F$ is a homeomorphism onto its image. On the other hand, we have Theorem III. There exists a continuous Lebesgue measure preserving function of the unit square onto itself which is not a homeomorphism. (Received November 9, 1961.)

587-61. P. M. COHN, Box 2155 Yale Station, New Haven, Connecticut. Factorization in non-commutative power series rings.

If a and b are formal power series in a single variable, with orders $o(b) \leq o(a) < \infty$, then there is a power series q such that $o(a - bq) > o(a)$. This property is generalized in the ring R of power series in any number of noncommuting indeterminates (over a field) and is used to prove the following unique factorization theorem for R : Any element of R which is neither zero nor a unit can be written as a finite product of irreducible factors, which are unique up to right and left multiplication by a unit. (Received November 10, 1961.)

587-62. D. R. ANDERSON, Duke University, Durham, North Carolina. On nested ordinal recursive functions and a subrecursive hierarchy.

Theorem 1. Let $<_R, <_S$ be two well-orderings. If $p(x)$ is a nested $<_S$ -ordinal recursive function such that if $x <_R y$ then $p(x) <_S p(y)$, then the nested $<_R$ -ordinal recursive functions are nested $<_S$ -ordinal recursive. O' is the set of primitive recursive ordinal notations. For a in O' , C_a is the class of functions defined by Kleene. (Extension of an effectively generated class of functions by enumeration, Colloq. Math. vol. 6 (1958) pp. 67-78.) Theorem 2. Let $<_R$ be a primitive recursive well-ordering. $f(x)$ is a nested $<_R$ -ordinal recursive function which may be defined from primitive recursive functions and one application of the scheme for defining nested $<_R$ -ordinal recursive functions. $f(x)$ is in C_a for some a in O' , $|a| = \omega < |R|$, and a depends only on $<_R$. Theorem 3. If $f(x)$ is in C_a for some a in O' , then $f(x)$ is nested $<_R$ -ordinal recursive for a primitive recursive well-ordering $<_R$ depending only on a and $|R| \leq \omega^{|a|+2}$. Theorem 4. For any ordinal number α there is a primitive recursive well-ordering $<_R$ such that $|R| = \alpha$ and the nested $<_R$ -ordinal recursive functions are primitive recursive. Theorem 5. If a is in O' and $|a| \neq \alpha + n$ and $|a| \neq \alpha + \omega$ then there is a b in O' such that $|b| = |a|$ and C_a is not contained in C_b . (Received November 10, 1961.)

Representation of an isotropic diffusion as a skew product.

The processes considered have state space R^3 and (i) possess the simple Markov property, (ii) are homogeneous in time, (iii) have continuous paths, (iv) are isotropic, (v) do not pass through the origin at positive times except on a set of zero probability. The sample space is the Cartesian product $\Omega \times \Omega'$. Ω and Ω' consist, respectively, of all continuous paths ω and ω' on $[0, \infty)$ and the unit sphere S^2 . The results obtained are: (a) The radial process $r(t, \omega)$ is simple Markov and homogeneous in time. (b) The whole process can be represented as a "skew" product of $r(t, \omega)$ and an independent spherical Brownian motion run with a clock $\sigma(t, \omega)$ depending on the radial path, (c) $\sigma(t, \omega)$ is non-negative nondecreasing continuous function of t for fixed ω . For fixed t it is measurable with respect to the subsigmafield determined by the radial motion up to time t . If ω_s^+ is defined by $r(t, \omega_s^+) = r(t + s, \omega)$, then with probability one for all pairs $s < t$ simultaneously it holds: $\sigma(t, \omega) = \sigma(t - s, \omega_s^+) + \sigma(s, \omega)$. (Received November 10, 1961.)

Complementary manifolds in matrix algebras.

The linear space M of all real n by n matrices $B = \{b_{ij}\}$, $C = \{c_{ij}\}$, ... is regarded as an n^2 dimensional Euclidean space, with the inner product $(B, C) = \sum \sum b_{ij}c_{ij}$. The set of all matrices of each of several types is a linear subspace of M ; the dimensions of these subspaces are determined, and it is shown that they decompose M in various ways into a direct sum of orthogonal complements. For example, the linear subspace H of all matrices whose row- and column-sums are zero (RCSZ matrices) is of dimension $(n - 1)^2$, and its orthogonal complement $H^\perp$ is the direct sum of mutually orthogonal subspaces X_0, X_1, X_2 , of respective dimensions $(n - 1)/2, 1, (n - 1)/2$. Also $H = G \oplus F$, where G is the subspace of symmetric, and F the subspace of antisymmetric, RCSZ matrices; the subspace G and the subspace D of diagonal matrices are nonorthogonal complements in the subspace S of all symmetric matrices. If P is any permutation, the matrix PBP^{-1} has rows and columns which are those of B in a symmetrical rearrangement; thus M and H are partitioned into congruent wedges. Canonical forms as representatives in one of the wedges are determined, boundary hypersurfaces and extreme points are discussed, and stochastic matrices are classified. A study is made also of rearrangements and the wedges corresponding to PBQ , where P and Q are permutations. (Received November 10, 1961.)

semigroups.

Using a theorem of S. T. Hu (Amer. J. Math. vol. 75 (1953) pp. 60-78) we observe: If G and K are compact connected groups and $f: G \rightarrow K$ is an epimorphism then $f^*: H^*(K) \rightarrow H^*(G)$ is a monomorphism. ($H^*(X)$ represents the Čech cohomology group of X with coefficients in the reals.) From this and a remark of A. D. Wallace (Summa Brasil. Math. vol. 3 (1953) pp. 43-54) we prove the following theorem: Let $(S, \cdot)$ be a compact connected semigroup with unit. If the topology of S admits some group structure (S, θ) then $(S, \cdot)$ is a group. (Received November 10, 1961.)

587-66. OTTO KOERNER, University of Utah, Salt Lake City, Utah. On mean values of trigonometrical sums in algebraic number fields.

Vinogradov's mean value theorem on trigonometrical sums with polynomial exponents is extended to algebraic number fields. The following applications are discussed. (i) Improved estimations of trigonometrical sums with respect to Siegel's general Farey-dissection in number fields. (ii) Information about the magnitude of certain mean values which play an important role in Waring's problem. It is possible to derive from (i) and (ii) an asymptotic formula for the number of representations of a totally positive algebraic integer as a sum of integral polynomial values. This formula specialized to the ordinary Waring problem leads to refinements of theorems previously proved by Siegel. (Received November 10, 1961.)

587-67. W. B. JURKAT, Syracuse University, Syracuse 10, New York, and ALEXANDER PEYERIMHOFF, University of Utah, Salt Lake City, Utah. On equivalence theorems of the Knopp-Schnee-Hausdorff type.

The methods of Cesàro and Hölder may be obtained by iteration processes from the arithmetic mean C_1 . These processes amount to defining powers C_1^α ($\alpha > -1$) of C_1 or powers of the "numerator" of C_1 . Similar iteration processes can be defined, e.g., for any positive triangular method, and the question arises whether the generalized Cesàro and Hölder methods obtained in this way are equivalent. The paper gives an explicit formula for the powers A^α of a matrix. It is shown that, for matrices which satisfy a certain mean value theorem plus some other fairly natural conditions, an inclusion theorem holds for the Cesàro and Hölder iterations with an order between 0 and 1. There is even equivalence in many cases, e.g., for all Nörlund means. (Received November 10, 1961.)

587-68. M. A. HYMAN, IBM Systems Center, 7220 Wisconsin Avenue, Bethesda 14, Maryland. Finding zeros of general functions automatically.

An algorithm is presented for finding as many zeros as desired of $F(x)$, where x, F are real and restrictions on $F(x)$ are slight; in particular, F need not be known as an explicit function of x . The procedure, a generalization of "false position," is fast and cannot diverge. Good "starting values" are not needed, but can be exploited if available. The algorithm seems well-suited to automatic computation, handling a wide variety of functions by the same rather simple logic. The procedure has been coded for the IBM 709 computer and applied successfully to a variety of functions - having poles, essential singularities, and "zeros at infinity." The algorithm can be extended to the cases (a) x, F are complex; (b) x, F are vectors. (Received November 10, 1961.)

587-69. J. V. RYFF, Stanford University, Stanford, California. Ordinary differential equations determined by contracting vector fields. Preliminary report.

Let $N(B)$ be a neighborhood of a compact subset of E^n , $I = [a, b]$ an interval of the real line, and $\phi(t; x)$ a vector field defined on $I \times N(B)$ taking values in E^n . Moreover, let ϕ satisfy conditions sufficient to insure a unique solution of the vector differential equation $x' = \phi(t; x)$ with initial vector

$x(a) = x_0$ in $N(B)$. What further property must ϕ have in order to guarantee that every solution $x(t)$ of the differential equation which enters B remains thereafter in B ? In his study of semigroups of transformations arising from certain infinitesimal transformations, C. Loewner (Math. Z. vol. 72 (1959) pp. 53-60) encounters a problem of this nature with differential equation of the form $x' = A(t)x$ and B a convex polyhedron. By showing that $A(t)x$ satisfies a certain contraction property on the boundary of B , it is possible to demonstrate that all solutions entering B remain in B . The contraction property is extended to arbitrary vector fields $\phi(t;x)$. Theorem. A necessary and sufficient condition that every solution $x(t)$ of $x' = \phi(t;x)$ with initial vector in $N(B)$ remain in B whenever it enters B is that ϕ satisfy the contraction property. (Received November 3, 1961.)

587-70. J. H. CURTISS, Box 8052, University of Miami, Coral Gables 46, Florida. Interpolation by harmonic polynomials.

Let $H_n(u;z)$ denote the harmonic polynomial of degree at most n found by interpolation in $2n + 1$ points to a function u given on the boundary C of a region D of the complex z -plane. It is proved that (a) for any bounded D there always exist interpolation points on C so that H_n can be uniquely determined for each n , and (b) for a wide class of Jordan regions D and for boundary data u with a smooth first derivative on C the points of interpolation on C can be chosen so that $\lim_{n \rightarrow \infty} H_n(u;z)$ exists, z on $C + D$, and gives the solution of the Dirichlet problem for u and D . Explicit formulas are derived for H_n in the case of interpolation on a circle and on an ellipse and convergence is proved in these cases for arbitrary continuous boundary data. Various generalizations are indicated. (Received November 13, 1961.)

587-71. K. W. KWUN and F. A. RAYMOND, University of Wisconsin, Madison, Wisconsin. Factorization of cells.

Poenaru, Mazur and Curtis have given examples of spaces X such that $X \times I = I^{n+1}$, where $X \neq I^n$, $n \geq 4$. All known examples have been combinatorial manifolds with boundary. In this paper, examples are given where X fails to be a manifold with boundary for each $n \geq 4$. The space is constructed by shrinking a wild arc in $\text{Int } I^{n-1}$ to a point and then multiplying by an arc. That X has the desired property is proved by showing that $X \times I$ is a manifold with the sphere boundary imbeddable in S^{n+1} . An essential use is made of the recent result of Andrews and Curtis on n -space modulo an arc. To facilitate the arguments a factorization theorem for generalized cells is first proved. (Received November 13, 1961.)

587-72. E. A. WALKER, J. M. IRWIN, and CAROL PEERCY, New Mexico State University, Box 396, University Park, New Mexico. On the splitting of Abelian groups.

Let G be an Abelian group. A subgroup H of G is called a high subgroup of G iff H is maximal disjoint from $G^1 = \bigcap_{n=1}^{\infty} I^n G$. A group G is said to split iff the torsion subgroup G_t of G is a direct summand of G . The relation between the splitting of high subgroups of G and the splitting of G is investigated. The main result obtained is the following. If G is reduced then G splits iff G/G_t is reduced and some high subgroup of G splits. In a sense, this reduces the splitting problem for reduced

groups to the case when the group has no elements of infinite height (since high subgroups have no elements of infinite height). A group G may split however without all high subgroups of G splitting. (Received November 13, 1961.)

587-73. R. C. GILBERT and V. A. KRAMER, University of California, Riverside, California. Trace formulas for a perturbed operator.

Theorem 1. Let T_1 and V be self-adjoint operators on a Hilbert space, with V bounded and $T_2 = T_1 + V$. If for some σ , $\| |V|^{1/2} R_1(z) \|_2 = O(\tau^{-\alpha})$ as $\tau \rightarrow \infty$, where $\alpha > 1/2$, $z = \sigma + i\tau$, $R_1(z)$ is the resolvent of T_1 , the norm is the Schmidt norm, then $\lim_{\tau \rightarrow \infty} \tau^2 S\{R_2(z) - R_1(z)\} = -\lim_{\tau \rightarrow \infty} \tau^2 S\{R_1(z) V R_1(z)\}$. Here $S\{A\}$ is the trace of A . Theorem 2. If T_1 is a self-adjoint extension of an ordinary differential operator L with spectral function ρ and if V is the operator of multiplication by q , then under certain integrability conditions $\lim_{\tau \rightarrow \infty} \tau^2 S\{R_1(z) V R_1(z)\} = -\int_{-\infty}^{\infty} \sum_{j,k=1}^n (V s_k(\lambda), s_j(\lambda)) d\rho_{jk}(\lambda)$. Here the s_k form a basis for the solutions of $Lu = \lambda u$. The theorems are applicable if T_1 is $-d^2/dx^2$ on $[0, \infty)$ with boundary condition $u(0) = 0$; from them is obtained $-\sum_{i=1}^m k_i^2 + (2/\pi) \int_0^{\infty} s \alpha(s) ds = -q(0)/4$ relating the eigenvalues $-k_i^2$ and asymptotic phase $\alpha(s)$ of T_2 to the potential q . (Received November 13, 1961.)

587-74. G. G. BILODEAU, Boston College, Chestnut Hill 67, Massachusetts. The summability of series of Hermite polynomials.

Let $H_n(x)$ be the Hermite polynomial of order n defined by $H_n(x) = (-1)^n e^{x^2} D^n e^{-x^2}$. If $a_n = \pi^{-1/2} \int_{-\infty}^{+\infty} e^{-x^2} H_n(x) f(x) dx$ then $f(x) \sim \sum_{n=0}^{\infty} a_n (2^n n!)^{-1} H_n(x)$. Under fairly severe restrictions on the function $f(x)$ in the neighborhoods of $\pm \infty$ and x , this series will converge to $f(x)$. Under less restrictive conditions, it has been shown to be summable to $f(x)$ by various methods of summability. The problem studied is to find a summability method which will sum the series to $f(x)$ under conditions on the function near $\pm \infty$ which do little more than assume the existence of a_n , $n = 0, 1, 2, \dots$. This problem is at least partially solved by the following theorem. Let summability (H) for a series $\sum_{n=0}^{\infty} u_n$ be defined by $\int_0^{\infty} e^{-t} \{ \sum_{n=0}^{\infty} u_n (\Gamma(n/2 + 1))^{-1} t^{n/2} \} dt$. Theorem. Let $\int_{-\infty}^{+\infty} e^{-(x-t)^2} |f(t)| dt < \infty$ for $-\infty < x < \infty$. If $f(x)$ is of B, V , in some neighborhood of x_0 , then the series defined above is summable (H) to $[f(x_0+) + f(x_0-)]/2$. (Received November 13, 1961.)

587-75. D. G. ARONSON, Stanford University, Stanford, California. On the stability of certain systems of parabolic difference equations.

Let $L \equiv P(x, t; D) - \partial/\partial t$ be an $N \times N$ matrix linear differential operator for $(x, t) \in E^n \times (0, T]$ $= R$. It is assumed that L is uniformly parabolic in the sense of Petrovskii in R . Let Λ be the difference operator obtained from L by replacing $(\partial/\partial x_j)u(x, t)$ by $(1/2h)\{u(x + e_j h, t) - u(x - e_j h, t)\}$ and $(\partial/\partial t)u(x, t)$ by $(1/\tau)\{u(x, t + \tau) - u(x, t)\}$, where $e_j = (\delta_{1j}, \dots, \delta_{nj})$. This paper shows that if Λ satisfies a stability condition which reduces to the condition given by F. John for a single equation of second order, then every solution of the initial value problem for Λ is bounded in terms of the initial data independent of h and $\tau = \lambda h^2$. The main tool of this investigation is an estimate for the fundamental solution of $\Delta u = 0$ in the case P independent of x . In particular, we show that the fundamental

solution of $\Delta u = 0$ has the same estimates as the fundamental solution of $Lu = 0$ (i.e., the Ladyženskaja estimates) provided the stability condition is satisfied. (Received November 13, 1961.)

587-76. D. C. RUNG, Pennsylvania State University, McAllister Building, University Park, Pennsylvania. Order of functions holomorphic in the unit disk.

Let D denote the unit disk. If $h(r)$ is a real-valued, continuous, increasing function, defined for all $r \geq 0$, with $h(0) = 0$, then a Carathéodory-type outer measure can be defined on the plane in terms of $h(r)$ in the usual fashion. Using a result of Lelong-Ferrand (Représentation conforme et transformations à intégrale de Dirichlet bornée, Paris, 1955, pp. 20 ff) it is proved that if $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is holomorphic in D with $\sum_{n=1}^{\infty} |a_n|^2 / n^s < \infty$, $s > -1$, then $(f^{(\beta)}(z))^2 (1-|z|)^{s+2\beta} [h(|z - e^{i\theta}|)]^{-1} \rightarrow 0$ as $z \rightarrow e^{i\theta}$, except possibly for a set of $e^{i\theta}$ of h -measure 0, where $f^{(\beta)}(z)$ is the generalized derivative of $f(z)$. Further, if $P(z)$ is holomorphic in D and omits there the values 0 and 1, then, for a $\mu > 0$ and $\epsilon > 0$, $|P(z)| \exp[-\mu(h(|z - e^{i\theta}|))^{1/2} (1-|z|)^{-1-\epsilon}] \rightarrow 0$ as $z \rightarrow e^{i\theta}$, except possibly for a set of $e^{i\theta}$ of h -measure 0. (Received November 13, 1961.)

587-77. JÖRAN FRIBERG, Box 2155 Yale Station, New Haven, Connecticut. A new classification of partially hypoelliptic differential operators.

A differential operator $P(D)$ is called partially hypoelliptic in a subset x' of the variables $x = (x', s'')$ if every solution of $P(D)u = 0$ is infinitely differentiable in the x' -variables. (Gårding and Malgrange, *Math. Scand.* vol. 9 (1961) pp. 5-21). This is the case if and only if $P(D) = P(D', D'') = Q(D') + \sum R_{\alpha}(D') D_{\alpha}'$, where, for some $b, c > 0$, $\sum_{\alpha} |Q^{(\alpha)}(\xi')| |\xi'|^a / b + \sum_{\alpha} |R_{\alpha}(\xi')| |\xi'|^c / c \leq C(|Q(\xi')| + 1)$ for all real ξ' . Then the numbers b and c determine the regularity properties of the solutions. In particular, if $Pu = 0$ and if u is in a nonquasianalytic class of Gevrey of order b/c in x'' then u is in a class of order b in x' . (Received November 13, 1961.)

587-78. R. M. McCONNEL, Duke University, Durham, North Carolina. Permutation polynomials over a finite field.

Let F_q be a finite field of order $q = p^n$. Let $m = (q-1)/d$, where d is an arbitrary divisor of $q-1$. For any $x \in F_q$, let $\psi_d(x) = x^m$. The first theorem is a generalization of a result due to L. Carlitz [*Proc. Amer. Math. Soc.* vol. 11 (1960) pp. 456-459]. Theorem 1. Let λ be a fixed element of F_q satisfying $x^d = 1$. Let $f(x)$ be any polynomial with coefficients in F_q such that $\psi_d(f(x) - f(y)) = \lambda \psi_d(x - y)$ for any $x, y \in F_q$. Then $f(x) = ax^{p^i} + b$, for some i in the range $0 \leq i < n$. Moreover $\psi_d(a) = \lambda$. Theorem 2. Let $m_1 = (q-1)/d_1$, $m_2 = (q-1)/d_2$, where d_1 and d_2 are any divisors of $q-1$. Let $\psi_1(x) = x^{m_1}$ and $\psi_2(x) = x^{m_2}$ for all $x \in F_q$. Let λ and μ be fixed elements of F_q such that $\lambda^{d_1} = 1$ and $\mu^{d_2} = 1$. Suppose $f(x, y)$ is a polynomial in x and y with coefficients in F_q satisfying $\psi_1(f(x, y) - f(z, y)) = \lambda \psi_1(x - z)$ and $\psi_2(f(x, y) - f(x, z)) = \mu \psi_2(y - z)$ for all $x, y, z \in F_q$. Then $f(x) = ax^{p^i} + by^{p^j} + c$ for some i and j such that $0 \leq i < n$ and $0 \leq j < n$. Moreover $\psi_1(a) = \lambda$ and $\psi_2(b) = \mu$. Theorem 2 has been extended to polynomials in k variables with a similar result. (Received November 13, 1961.)

587-79. A. A. KOSINSKI, University of California, Berkeley 4, California. Piecewise linear real functions on combinatorial manifolds.

This paper considers piecewise linear mappings of combinatorial manifolds into the real line. There is defined - in a purely combinatorial way - a certain class of mappings called nondegenerate and it is proved that every continuous mapping may be approximated arbitrarily closely by a nondegenerate one. Nondegenerate mappings on a combinatorial manifold behave like differentiable nondegenerate mappings on a differentiable manifold. In particular, one can define the index of a singularity and then prove Morse inequalities. We prove also that if on a closed manifold M there exists a piecewise linear real function with only two critical nondegenerate singularities then M is combinatorially equivalent to S^n . The approximation theorem is also extended to height function on combinatorial submanifolds of the Euclidean space. (Received November 13, 1961.)

587-80. R. M. BLUMENTHAL, Institute for Advanced Study, Princeton, New Jersey and R. K. GETTOOR, University of Washington, Seattle, Washington. Markov processes equivalent under a change of time.

Let $X(t)$ and $Y(t)$ be regular time-homogeneous Markov processes in a locally compact separable metric space E . Let $M_X(N_X)$ be the expectation operators for the process X (Y) and let $S_U(T_U)$ be the time X (Y) hits the subset U of E . Suppose that for each starting point x and compact U , $X(S_U)$ has the same distribution as $Y(T_U)$. Suppose further that one can write $E = \bigcup G_n$ with G_n open and $\bar{G}_n \subset G_{n+1}$ where $N_X(T_{G_n^c})$ is a bounded function approaching 0 as x approaches the boundary of G_n and where $M_X(S_{G_n^c}) < \infty$. (Here c denotes complement.) Then there is a continuous strictly increasing additive functional $\phi(t)$ of X such that for each x in E and bounded neighborhood U of x , $M_X(\phi(S_{U^c})) = N_X(T_{U^c})$. Thus, if $b(t)$ denotes the random time change inverse to $\phi(t)$ then the process $X(b(t))$ has the same extended infinitesimal operator (as defined by Dynkin) as the process $Y(t)$. (Received November 15, 1961.)

587-81. CARL FAITH, Institute for Advanced Study, Princeton, New Jersey. Johnson and Goldie prime rings.

A r -Johnson ring $(R, *)$ is a prime ring R with a map $I \rightarrow I^*$, I any r -ideal of R , satisfying (P1)-(P7) of R. E. Johnson's [Trans. Amer. Math. Soc. vol. 72 (1953) pp. 351-357]. (1) For each r -ideal I of $(R, *)$, $I^* \subseteq I^{\ell r} = r$ -annihilator of the ℓ -annihilator of I . A Johnson ring $(R, *, *)$ is a ring R which is both r - and ℓ -Johnson. (2) If $(R, *, *)$ satisfies the maximum condition on annihilator r -ideals, then $I^* = I^{\ell r}$, for each r -ideal I . A ring satisfying the right quotient (r.q.) and l.q. conditions is a Goldie ring. The author proved (Abstract 576-190, Notices Amer. Math. Soc. vol. 7 (1960) p. 966) that every Goldie prime ring is a Johnson ring with $I^* = I^{\ell r}$, $J^* = J^{\ell r}$, for each r -ideal I , and each ℓ -ideal J . (3) If $(R, *, *)$ has finite r -dimension (in Goldie's sense), then R satisfies the r.q. conditions. (4) If R is prime with finite ℓ - and r -dimensions, then R is a Johnson ring if and only if it is a Goldie ring. Let $A = D_n$ be the ring of $n \times n$ matrices over a division ring D . A subring R is a (left) D -subring if to each $a \in A$ there exists $0 \neq d \in D$ (scalar matrix) such that $da \in R$. (5) If R is a Goldie prime ring, then R is a D -subring of its quotient ring $A = D_n$, D varying over all S such that

$A = S_n$. (6) If R has finite ℓ -dim, and is a D -subring (for a fixed D) of $A = D_n$, $n > 1$, then R satisfies the ℓ .q. conditions, and A is the ring of left quotients of R . (Received November 15, 1961.)

587-82. R. E. FULLERTON, University of Maryland, College Park, Maryland. Geometric structure of conditional basis systems in a Banach space.

Let X be a real Banach space with conjugate X^* . A sequence of pairs of elements $\{(x_n, x_n^*)\}$, $n = 1, 2, 3, \dots$, $x_n \in X$, $x_n^* \in X^*$ is a biorthogonal basis system for X if for each n , $x_n^* x_m = \delta_{nm}$ and for each $x \in X$, $x = \sum_{n=1}^{\infty} x_n^*(x) x_n$ where this expansion is unique. The basis is conditional if for some $x \in X$, the convergence of the expansion of x is conditional. It is shown that if x is an element with a conditional expansion, then any rearrangement of the series diverges or converges to x . It is also shown that a necessary and sufficient condition that X have a conditional basis system is that there exist in X a cone K with extreme rays $\{r_n\}$, $n = 1, 2, 3, \dots$, such that (1) for each n the closed linear space L_n determined by all $\{r_k\}$ with $k \neq n$ is a hyperplane, (2) $K = \bigcap_{n=1}^{\infty} H_n$ where H_n is the half space bounded by L_n which contains r_n , (3) for some $x \in K$, $P_x = K \cap (x - K)$ has a maximal nested sequence of faces $\{F_k\}$ each of finite deficiency such that if u_n is the maximal element of F_n in the order defined by K , then the sequence $\{u_n\}$ converges to zero but the faces F_n have diameters bounded away from zero. (Received November 15, 1961.)

587-83. P. C. HAMMER, Department of Numerical Analysis, University of Wisconsin, 5534 Sterling Hall, Madison 6, Wisconsin. Extended topology: Generation of subadditive and additive subfunctions.

Let $\bar{M}$ be the class of all subsets of a space M with null set N . Let F be the family of all functions mapping $\bar{M}$ into itself. Then $f(X \cup Y) \subseteq fX \cup fY$, $f(X \cup Y) \supseteq fX \cup fY$, $f(X \cup Y) = fX \cup fY$ respectively define properties of subadditivity, isotonicity, and additivity of $f \in F$ when the relations hold for all $X, Y \in \bar{M}$. Three interior transformations, t_i , mapping F into itself are defined such that $t_0 f = f$ if and only if f is subadditive, $t_1 f = f$ if and only if f is isotonic, and $t_2 f = f$ if and only if f is additive. Moreover, for any $f \in F$, $t_0 f$, $t_1 f$, $t_2 f$ are respectively the unique maximal subadditive, isotonic, and additive functions contained in f . Thus, $t_0 f$ is the maximal additive closure function contained in a closure function. Dualization yields the minimal superintersective and intersective functions containing a given function. Related considerations apply to real-valued set-functions. (Received November 15, 1961.)

587-84. A. L. LIULEVICIUS, The Institute for Advanced Study, Princeton, New Jersey. The stable symplectic group and the cohomology of an algebra.

Let A be the Steenrod algebra over Z_2 , A^* the graded dual of A . Let $\alpha: A \rightarrow A$ be the dual of the squaring map in A^* . Consider the following elements of A : $a_i = Sq^{2^i}$, $Q_0 = Sq^1$, $Q_{i+1} = [a_{i+1}, Q_i]$, $R_0 = Sq^2$, $R_{i+1} = [a_{i+2}, R_i]$. Let B be the subalgebra of A generated by 1 , Q_i , R_j , $i, j = 0, 1, \dots$; let C be the subalgebra generated by 1 and Q_i , $i = 0, 1, \dots$ alone. Let $\beta = \alpha|_B$; then Kernel $\beta = C$ and Image $\beta = C$. A minimal resolution of Z_2 over C is easily constructed; a free resolution of Z_2 over B is constructed as a twisted tensor product of two such minimal resolutions over C . The structure of

$\text{Ext}_B(Z_2, Z_2)$ is exhibited. This determines the E_2 -term of the Adams spectral sequence for the stable homotopy of the spectrum of Thom spaces for the symplectic groups (see also S. P. Novikov, Dokl. Akad. Nauk SSSR vol. 132 (1960) pp. 1031-1034). Results on homotopy will be presented. (Received November 15, 1961.)

587-85. MARVIN MARCUS, University of British Columbia, Vancouver 8, B. C. Canada and MORRIS NEWMAN, Division 11.1, National Bureau of Standards, Washington 25, D. C. The permanent of a symmetric matrix.

A doubly stochastic matrix S has non-negative entries and every row and column sum is one. The permanent of an n -square matrix A is defined by $\text{per}(A) = \sum_{i=1}^n \prod a_{i\sigma(i)}$, in which the summation extends over all permutations σ of $1, \dots, n$. In a recent announcement (The permanent function as an inner product, Marvin Marcus and Morris Newman, Bull. Amer. Math. Soc. vol. 67 (1961) pp. 223-224) it was proved that (*) if A is n -square doubly stochastic and symmetric positive semi-definite then $\text{per}(A) \geq n!/n^n$ with equality if and only if $A = J_n$, the matrix all of whose entries are $1/n$. In the present paper the following extension of this result is obtained: (**) if A is n -square symmetric positive semi-definite and $a_{ij} \geq s_{ij}$, $i, j = 1, \dots, n$ where $S = (s_{ij})$ is doubly stochastic then $\text{per}(A) \geq n!/n^n$. The equality holds if and only if $A = DJ_nD$ where D is a diagonal matrix and $\text{per} D = 1$. This result does not follow directly from (*) since simple examples show that the hypotheses in (**) do not imply that there exists a symmetric positive semi-definite doubly stochastic S such that $a_{ij} \geq s_{ij}$, $i, j = 1, \dots, n$. A special implicit function result together with an investigation of the extrema of a function defined on the polyhedron of doubly stochastic matrices are required to reduce (**) to (*). (Received November 15, 1961.)

587-86. DAVID RYEBURN, Ohio State University, Columbus 10, Ohio. Additivity of topological properties.

Nagata (J. Inst. Polytech. Osaka City Univ. Ser. A vol. 1 (1950) pp. 93-100) showed that the union of a locally finite family of closed metrizable spaces is metrizable. A new proof of this result is given, using results of De Groot and Smirnov. Similar additivity theorems are proved for various separation, compactness, and countability properties, for open and for closed coverings satisfying suitable finiteness or countability conditions. Many of the proofs use the Tietze Extension Theorem or similar functional characterizations. For closed coverings local finiteness is frequently a sufficient condition for additivity. For open coverings the results are mainly negative, but the situation improves if the whole space is assumed to be normal or paracompact. (Received November 15, 1961.)

587-87. W. T. SLEDD, Michigan State University, East Lansing, Michigan. Summability of ordinary Dirichlet series by Perron-type matrices.

Let $f(z)$ be analytic for $|z| < R$ where $R > 1$ and suppose that $f(1) = 1$. Set $[f(z)]^n = \sum_{k=0}^{\infty} f_{kn} z^k$. Then the matrix $F = (f_{kn})$ determines a series-to-series transformation and is called a Perron-type matrix. Set $D(s) = \sum_{n=0}^{\infty} u_n (n+1)^{-s}$. Then additional conditions are imposed on $f(z)$ and $\{u_n\}$ in order to insure that (a) F summability of $D(s_0)$ implies F summability of $D(s)$ for $\text{Re}(s) > \text{Re}(s_0)$, (b) F sum-

mability of $D(s_0)$ implies $|F|$ summability of $D(s)$ for $\text{Re}(s) > \text{Re}(1 + s_0)$, and (c) $|F|$ summability of $D(s_0)$ implies $|F|$ summability of $D(s)$ for $\text{Re}(s) > \text{Re}(s_0)$. (Received November 15, 1961.)

587-88. H. E. RICHERT, Syracuse University, Syracuse 10, New York. On some connections between Mertens' hypothesis and Dirichlet's L-series.

Let $\mu(n)$ denote Möbius function, and let $M(x) = \sum_{n \leq x} \mu(n)$. It was conjectured by Mertens that $|M(x)| \leq x^{1/2}$. Recently, Jurkat developed a method to prove that $\limsup |M(x)|/x^{1/2} > 1/2$. It can be shown that his method can be extended to give connections between the function $M(x)$ and certain questions regarding Dirichlet's L-series. In particular, a formula connecting Mertens' conjecture and the class number in imaginary quadratic fields is given. (Received November 15, 1961.)

587-89. JOSEPH AUSLANDER, RIAS, 7212 Bellona Avenue, Baltimore 12, Maryland. Endomorphisms of minimal sets. Preliminary report.

Let (X, T) be a minimal set (X compact Hausdorff). Let $H(X)$ be the semigroup of endomorphisms of (X, T) (that is, continuous maps of X to X which commute with the action of T), and let $A(X)$ be the group of automorphisms of (X, T) . If $A(X) = H(X)$, (X, T) is called coalescent. Let E be the enveloping semigroup of (X, T) , and let I be a minimal right ideal in E . Then (X, T) is isomorphic with $(I/R, T)$, where R is a closed T -invariant equivalence relation in I . Theorem 1. $(I/R, T)$ is coalescent if and only if for any $p \in I$, pR is not a proper subset of R . We define a quasi-ordering $>$ in X by $x > y$ if (i) $xp = x$ implies $yp = y$, and (ii) $xq = xq'$ implies $yq = yq'$ ($p, q, q' \in I$). Theorem 2. There exists a (necessarily unique) $\phi \in H(X)$ with $x\phi = y$ if and only if $x > y$. Let S denote the equicontinuous structure relation of (X, T) , and let $\pi: X \in X/S$ be the natural projection. Define $\pi^*: H(X) \rightarrow H(X/S)$ by $(x\pi)(\phi\pi^*) = (x\phi)\pi$. Then π^* is well defined and is a homomorphism. Theorem 3. Suppose that T is abelian, and (X, T) is locally almost periodic. Then π^* is one-to-one, and therefore $H(X)$ is abelian. If π^* is onto, (X, T) is coalescent. (Received November 16, 1961.)

587-90. J. BURLAK, North Carolina State College, Raleigh, North Carolina. On the solution of certain dual integral equations.

Dual integral equations of the form $\int_0^{\infty} u^a \psi(u) J_\nu(xu) du = f(x)$, $0 \leq x < 1$, $\int_0^{\infty} \psi(u) J_\nu(xu) du = g(x)$, $x > 1$, where $\psi(x)$ is the unknown function and $f(x)$ and $g(x)$ are given, have applications to mixed boundary-value problems in partial differential equation. Solutions have been given in the special cases $g(x) \equiv 0$ (the Titchmarsh case), $f(x) \equiv 0$ and also in the general case. It is shown that the solution in the general case is obtained more easily if the equations are first transformed to the Titchmarsh form, so that this is the only case that needs explicit solution. A similar transformation allows the solution of the more general equations $\int_0^{\infty} u^a \{1 + H(u)\} \psi(u) J_\nu(xu) du = f(x)$, $0 \leq x < 1$, $\int_0^{\infty} \psi(u) J_\nu(xu) du = g(x)$, $x > 1$, where $H(u)$ is given, to be reduced to the problem of solving a Volterra integral equation of the second kind. (Received November 16, 1961.)

587-91. R. F. DeMAR, Miami University, Oxford, Ohio. Existence of interpolating functions of exponential type.

Let K be the set of entire functions of exponential type. Let $D(f)$ denote the conjugate indicator diagram of f in K . If C is a simply connected set of complex numbers, let $K[C]$ denote the set of all $f \in K$ such that $D(f) \subseteq C$. Let $\{\mathcal{L}_n\}$ be a sequence of linear functionals defined on K . For a sequence $\{b_n\}$ of complex numbers, if there exists f in $K[C]$ such that $\mathcal{L}_n(f) = b_n$ for each n , then $\{b_n\}$ is said to be admissible for $\{\mathcal{L}_n\}$ and $K[C]$. Buck (Duke Math. J. vol. 15 (1948) pp. 879-891) gave a necessary condition for admissibility of a sequence $\{b_n\}$ which applies to a general class of sequences $\{\mathcal{L}_n\}$ of linear functionals and corresponding uniqueness classes $K[C]$ of functions. In this paper, it is shown that if C is a convex set containing the origin, this condition is also sufficient. Example of an application: A sequence $\{b_n\}$ is admissible for the Stirling functionals $\mathcal{L}_n = \Delta^n f(-n/2)$ and $K[a, c]$, $c < \pi$ if and only if the function $b(z) = \sum b_n z^n$ is regular at the origin and can be continued to the interval $[-i/2, i/2]$ of the imaginary axis. (Received November 16, 1961.)

587-92. B. A. FUSARO, University of Oklahoma, Norman, Oklahoma. Does the surface-measure of a sphere depend on its location in space?

By "sphere" is meant "geodetic sphere", and by "space" is meant "Riemannian space". The answer is evidently "Yes", unless some restriction is placed on the space. In a two-dimensional space of constant positive curvature, the answer is easily seen to be "No". It seems to follow quite simply from a result of Lichnerowicz (1953) that in a harmonic space the answer is also "No". From a result of Copson and Ruse (1939-1940) it then follows that, in particular, the answer is "No" for any dimensional space of constant positive curvature, an intuitively reasonable result. (Received November 16, 1961.)

587-93. SIMON HELLERSTEIN, Stanford University, Stanford, California. The real zeros of three successive derivatives of an entire function.

Let $f(z)$ be entire. Denote by $n(r, 1/f)$ the number of zeros of $f(z)$ (taking multiplicities into account) which lie in the disk $|z| \leq r$, and by $T(r, f)$ the Nevanlinna Characteristic of f . Assume that all but a finite number of the zeros of f, f' and f'' lie on the real axis. Then: (a) $k \geq 3$ and $\limsup_{r \rightarrow \infty} \{\log_k n(r, 1/f) / \log r\} = \lambda$ imply $\limsup_{r \rightarrow \infty} \{\log_k T(r, f) / \log r\} = \lambda$ (k an integer, and $\log_k(x)$ denotes the k -fold iterated logarithm of x); (b) $\limsup_{r \rightarrow \infty} \{\log_2 n(r, 1/f) / \log r\} = \lambda$ implies $\limsup_{r \rightarrow \infty} \log_2 T(r, f) / \log r \leq \max(1, \lambda)$. Assertion (b) with $\lambda = 0$ contains Theorem 3 of Edrei (Trans. Amer. Math. Soc. vol. 78 (1955) pp. 276-293). More generally it is shown that if all but a "small" number of the zeros of f, f', f'' lie on the real axis, then the growth of $T(r, f)$ is restricted by that of $n(r, 1/f)$ provided r avoids the values of an exceptional set of finite measure. (Received November 16, 1961.)

587-94. G. S. JONES, RIAS, 7212 Bellona Avenue, Baltimore 12, Maryland. Quasi-sinusoidal solutions of nonlinear differential-difference equations.

Consider functional differential equations of the form: (1) $x'(t) = F(a, x(t), x(t-1))$, where a is a real parameter and $F(a, u, v)$ is continuous in a and u and Lipschitzian in v . Furthermore, assume $F(a, u, v)$ can be written $F(a, u, v) = f(a, v)g(u)$ where g is entire with a unique smallest real zero, $g(0) \neq 0$, $f(a, u) = au + O(u^2)$ as $u \rightarrow 0$, and $f(a, u) = 0$ iff $u = 0$. It can be shown that for a properly chosen (1) has a manifold M of periodic solutions of period $\lambda > 2$ with the following properties: (a) if $\psi \in M$ and $\psi(t_0) = 0$, then ψ has a single simple zero z_0 in $(t_0, t_0 + \lambda)$; (b) $\int_t^{t+\lambda} \psi(\tau) d\tau = 0$ for all real t ; (c) for $t \in (t_0, t_0 + \lambda)$, $\psi'(t) = 0$ iff $t = t_0 + 1$, or $t = z_0 + 1$. The proof involves showing the existence of a convex set S of continuous bounded initial functions defined on $[-1, 0]$ such that for each ϕ in S , $\phi(-1) = 0$, $\phi(x) \neq 0$, and the corresponding uniquely determined solution of (1) exist for all $t > 0$ and has at least two positive zeros. If $z_2(\phi)$ for ϕ in S denotes the second zero of the corresponding solution $x(\phi)$ of (1) then the operator T defined on S by the formula $T(\phi)(\tau) = x(\phi, z_2(\phi) + 1 + \tau)$, τ in $[-1, 0]$, can be shown to have a fixed point ϕ^* in S . The corresponding solution of (1) and all its translates are periodic solutions with the prescribed properties. (Received November 16, 1961.)

587-95. H. B. JENKINS and J. B. SERRIN, University of Minnesota, Institute of Technology, Minneapolis 14, Minnesota. Variational problems of minimal surface type.

A nonparametric variational problem $\delta \int F(p, q) dx dy = 0$ will be called of minimal surface type if it arises "essentially" from a regular parametric variational problem. Let $\phi(x, y)$ denote a solution of a variational problem of minimal surface type. Using results of one of the authors and the Bers-Nirenberg form of the Harnack inequality, a priori estimates of $|\nabla \phi|$ in terms of a bound from above on ϕ , and distance to the boundary are obtained. These estimates have the form $|\nabla \phi|_0 \leq C \exp(kM/R)$, where R is the distance from (x_0, y_0) to the boundary, $M = l.u.b. (\phi(x, y) - \phi(x_0, y_0))$, $|\nabla \phi|_0 = |\nabla \phi(x_0, y_0)|$, and C, k are constants depending only on the integrand $F(p, q)$. For the minimal surface equation this estimate becomes $|\nabla \phi|_0 \leq (e/a) \exp 2M/(1-a)R$, where $0 < a \leq 1/3$. The above estimates are applied to obtain a new proof of a theorem of R. Finn, namely the solvability of the boundary value problem for continuous data on convex curves. It is also shown that positive solutions satisfy a Harnack type inequality. The same techniques are used to improve and simplify estimates of the curvature of solutions of regular parametric variational problems given by one of the authors. (Arch. Rational Mech. Anal. vol. 8, no. 5 (1961)). (Received November 16, 1961.)

587-96. J. B. MUSKAT, 821 Cathedral of Learning, University of Pittsburgh, Pittsburgh 13, Pennsylvania. r as an r th power residue.

For r odd, let $\tau(x)$ denote the Gaussian sum $\sum_{n=1}^{p-1} x(n) \zeta_p^n$, where p is a prime $\equiv 1 \pmod{r}$, x is a primitive r th power Dirichlet character, modulo p , and $\zeta_p^p = 1$, $\zeta_p \neq 1$. Ankeny (Pacific J. Math. vol. 10 (1960) pp. 1115-1124) showed that if r is an odd prime, r is an r th power residue, modulo p , if and only if $\sum_j a_j - ra_{r-1}/2 \equiv 0 \pmod{r^2}$, where the a_j are defined by $\tau(x)^r = p \sum a_j \zeta_r^j$, $\zeta_r^r = 1$, $\zeta_r \neq 1$. (These and subsequent summations run from 1 to $r-1$.) Theorem. $\tau(x)^r + \tau(\bar{x})^r \equiv 2 + r(p-1) - 2r^2 + r^2(\chi(r) + \bar{\chi}(r)) \pmod{(1 - \zeta_r)^{2r+1}}$. Corollary 1. .5 is a fifth power residue,

modulo p , if and only if $a_1 + a_4 \equiv a_2 + a_3 \pmod{125}$. Corollary 2. If $r = 7$, $a_1 + a_6 \equiv a_2 + a_5 \equiv a_3 + a_4 \pmod{49}$. Corollary 3. $a_j \equiv 1 \pmod{r}$, $j = 1, \dots, r - 1$. Corollary 4. $p \sum a_j = r - 1 - r(p - 1)/2 \pmod{r^3}$. (Received November 16, 1961.)

587-97. C. J. NEUGEBAUER, Purdue University, West Lafayette, Indiana. Darboux functions of Baire class one and derivatives.

Let $I_0 = [0, 1]$ and let R be the reals. A function $f: I_0 \rightarrow R$ will be said to be in the class (B_1, D) iff f is of Baire class at most one and possesses the Darboux property. If Δ is the class of all $f: I_0 \rightarrow R$ which are derivatives, then $\Delta \subset (B_1, D)$. Theorem 1. $f \in (B_1, D)$ iff for each $a \in R$, the sets $\{x: f(x) \geq a\}$, $\{x: f(x) \leq a\}$ are G_δ and have compact components. Let $\{I\}$ be the collection of all non-degenerate compact subintervals of I_0 . Definition. $f: I_0 \rightarrow R$ is said to have property (C_1) iff for each $I \in \{I\}$ there is a point $x_1 \in I^0$ such that $I \rightarrow x$ implies that $f(x_1) \rightarrow f(x)$, $x \in I_0$. Theorem 2. $f \in (B_1, D)$ iff f has property (C_1) . Definition. $f: I_0 \rightarrow R$ is said to possess property (C_2) iff for each $I \in \{I\}$ there is a point $x_1 \in I^0$ such that (1) $I \rightarrow x$ implies $f(x_1) \rightarrow f(x)$, $x \in I_0$, and (2) $f(x_1)$, $|I|$ is an additive interval function on $\{I\}$. Theorem 3. $f \in \Delta$ iff f has property (C_2) . By Theorem 2, the condition (2) is precisely the property by which a function in (B_1, D) may fail to be a derivative. (Received November 16, 1961.)

587-98. R. W. McKELVEY, University of Colorado, Boulder, Colorado. The spectra of minimal self-adjoint extensions of a symmetric operator.

Let T be a closed symmetric operator with domain dense in a Hilbert space H . A (generalized) spectral resolution of T is a monotone increasing family E_μ of self-adjoint operators (μ real), with $E(-\infty) = 0$, $E(\infty) = 1$, and in terms of which (Tu, v) and $\|Tu\|^2$ have the usual representations as spectral integrals. M. A. Naimark has shown that to each family E_μ corresponds a self-adjoint extension T^+ of T in a Hilbert space $H^+ \supseteq H$ such that $E_\mu = P E_\mu^+$, where E_μ^+ is the spectral family of projections of T^+ and P is the orthogonal projection on H in H^+ . When E^+H is fundamental in H^+ , then T^+ is called a minimal extension of T . In this paper various aspects of the spectrum of a minimal extension T^+ are characterized in terms of E_μ and of the (generalized) resolvent $R_\lambda = \int_{-\infty}^{\infty} (\mu - \lambda)^{-1} dE_\mu$. It is shown that the Weyl and Aronszajn theorems on invariance of the essential and absolutely continuous spectra (of s.a. extensions in H of a given T , with equal finite defect numbers) hold for certain extensions T^+ in a larger space H^+ . Other extensions are seen to exhibit a very different character, first described for regular ordinary differential operators by E. A. Coddington and R. C. Gilbert [Trans. Amer. Math. Soc. vol. 93]. (Received November 16, 1961.)

587-99. BARTH POLLAK, Institute for Defense Analyses, 100 Prospect Avenue, Princeton, New Jersey. On the commutator subgroup of the orthogonal group over the 2-adic numbers.

Let V be a 4-dimensional vector space over Q_2 , the field of 2-adic numbers with a nondegenerate symmetric inner product. Let $O(V)$ be the orthogonal group of V , $O'(V)$ the subgroup of elements of determinant 1 and spinor norm 1 and $\Omega(V)$ the commutator subgroup of $O(V)$. Of course, if V is isotropic, $O'(V) = \Omega(V)$. This paper proves (in contrast to the case of Q_p , p odd) that $O'(V) = \Omega(V)$ when V is anisotropic also. (Received November 16, 1961.)

587-100. BEAUREGARD STUBBLEFIELD, Michigan State University Oakland, Rochester, Michigan. A sufficient condition for the imbedding of Peano continua in 1-manifolds.

A simple triod is a set of three arcs A_1 , A_2 , and A_3 such that, for $i \neq j$, $A_i \cap A_j = t$, an end point of A_i for each i . For any integer $n > 0$, G. S. Young (Bull. Amer. Math. Soc. vol. 50 (1944) p. 714) has defined a T_n -set to be the sum of an n -cell, G , and an arc, A , such that $G \cap A$ is a single point which is an end point of A and a relatively interior point of G . Thus, a T_1 -set is a simple triod. For any integer $n > 0$, an nT -set is a continuum which is the Cartesian product of a T_1 -set and $\mathbb{R}^{n-1}$. Space S is said to be of degree n if S can be imbedded in an n -manifold but cannot be imbedded in an $(n-1)$ -manifold. The author uses the fact that under a homeomorphism, limit points go into limit points to prove Theorem 1. The product of a T_j -set and a kT -set, $k \geq 1$, $j \geq 1$, cannot be imbedded in a $(j+k+1)$ -manifold. Theorem 2. If P is a Peano continuum such that P^n , $n \geq 2$, can be imbedded in an $(n+1)$ -manifold, then P can be imbedded in a 1-manifold. Corollary 1. $P^2 \subset M^3 \Rightarrow P \subset M^1$. Corollary 2. There exists no Peano continuum P whose square is of degree 3. (Received November 16, 1961.)

587-101. H. C. MILLER, JR., P. O. Box 3894, University, Alabama. A generalization of the Stieltjes transform. Preliminary report.

The classical notion of the Stieltjes transform of a bounded nondecreasing function is generalized from the real line to the boundary of a closed convex nonseparating body in the complex plane. An inversion formula and asymptotic properties analogous to those of the classical Stieltjes transform are proved and a necessary condition is given. (Received November 16, 1961.)

587-102. LOUIS SUCHESTON, 3702 N. Murry, Shorewood 11, Wisconsin. A zero-one property of mixing sequences of events.

A sequence of events $\{A_n\}$ is called mixing if for each event M , $P(A_n M) - P(A_n)P(M) \rightarrow 0$. A sequence of events $\{A_n\}$ is called zero-one if its tail- σ -field is trivial; semi-zero-one if every subsequence of $\{A_n\}$ admits a subsequence which is zero-one. Theorem. A sequence of events is mixing if and only if it is semi-zero-one. It is also shown under rather weak assumptions that mixing with a given limiting distribution function of a sequence of random variables [cf. Rényi, Acta Math. Acad. Sci. Hungar. (1958)] is invariant under change of measure. The proofs are based on the martingale theory. (Received November 16, 1961.)

587-103. HERMAN VAN ROSSUM, Michigan State University, East Lansing, Michigan. Classes of orthogonal polynomials with totally positive moment sequences.

For the sequence of real numbers $\{c_n\}_0^\infty$ define the determinant $\Delta_{\nu\mu}^{(c)}$ by $\Delta_{\nu\mu}^{(c)} = |c_{\nu-\mu+i+j}|_{i,j=0}^\mu$ ($\nu, \mu = 0, 1, 2, \dots$); $c_r = 0$ if $r < 0$. If $\Delta_{nn}^{(c)} \neq 0$ there exists a unique system of normed polynomials $B_n(x)$ ($n = 0, 1, 2, \dots$) in the indeterminate x , orthogonal with respect to the linear operator $\Omega^{(c)}$, where $\Omega^{(c)}(x^n) = c_n$ ($n = 0, 1, 2, \dots$), $\{c_n\}_0^\infty$ is said to belong to class H iff it satisfies the conditions $\Delta_{nn}^{(c)} \neq 0$; $\limsup |c_n|^{1/2} = \rho < \infty$. Theorem. The system of orthogonal polynomials associated with the sequence $\{c_n\}_0^\infty$ belonging to class H, is orthogonal on a circle $|z| = \rho + \epsilon$ ($\epsilon > 0$) and has a weight

function $\psi(z) = \sum_{n=0}^{\infty} c_n z^{-n-1}$. Let T be the class of sequences $\{c_n\}_0^{\infty}$ with $(-1)^{\mu(\mu+1)/2} \Delta_{\nu\mu}^{(c)} > 0$ (totally positive moment sequences). Then it holds $T \subseteq H$. The class of Bessel polynomials is a subclass of the class of orthogonal polynomials associated with T . The latter class is studied using the Padé table for the generating function of a sequence belonging to T (first considered by Edrei Canad. J. Math. vol. 5 (1953) pp. 86-94). Results are found for this class concerning coefficients asymptotic relations and location of zeros. The notion of true circle of orthogonality is introduced. (Received November 16, 1961.)

587-104. ALEXANDER HURWITZ, University of California, Los Angeles, California, and J. L. SELFRIDGE, University of Washington, Seattle 5, Washington. Fermat numbers and perfect numbers.

The Fermat numbers $F_m = 2^{2^m} + 1$ are prime for $0 \leq m \leq 4$ but no other Fermat primes are known. The prime or composite nature of these numbers was known for all $m \leq 4$ in 1640, 5 in 1732, 6 in 1880, 7 in 1905, 9 in 1909, 12 in 1952, 13 in 1960, and is now known for all $m \leq 16$. After 42 hours computing, the UCLA IBM 7090 found that F_{14} does not divide $1 + 3^{2^{16383}}$ and is therefore composite. The machine checked each step modulo $2^{35} - 1$. The even perfect numbers are $2^{p-1}(2^p - 1)$ where $2^p - 1$ is prime and no odd perfect numbers are known. Four perfect numbers were known in antiquity, 8 in 1772, 12 in 1914, 17 in 1952, 18 in 1957, and 20 are now known. The computer found that $2^{4253} - 1$ and $2^{4423} - 1$ divide the 4252nd and 4422nd terms, respectively, of the recurring series 4, 14, 194, 37634, ..., and are therefore prime. These are the largest prime numbers known. The machine tested each $2^p - 1$ with $p < 5000$ if no factor was known. (Received November 16, 1961.)

587-105. H. L. JOHNSON, 4801 North Dixie Drive, Dayton 14, Ohio. On the existence of a solution to a nonlinear wave equation with mixed boundary conditions.

The dynamics of a tethered balloon cable can be described by a nondimensional tension $w(x,y)$ and angle of inclination $\theta(x,y)$ where x denotes arc length along the cable and y -time. This is done through the equations (1) $w_{\theta_{xx}} - \theta_{yy} = -2w_x\theta_x$, (2) $w_{xx} = w\theta_x^2 - \theta_y^2$ and boundary conditions (3) $w_x(0,y) = \sin\theta(0,y)$, (4) $w\theta_x(0,y) = \cos\theta(0,y)$, (5) $w(1,y) = f_1(y)$, $\theta(1,y) = f_2(y)$, $\theta(x,0) = \theta_0(x)$, $\theta_y(x,0) = \theta_1(x)$, where f_1 , f_2 , θ_0 , and θ_1 are given and satisfy certain admissible conditions. These equations can be transformed into the form $\bar{u} = A(\bar{u})$ with $\bar{u} = (u,v,u_x, v_x,u_y,v_y)(x,y)$. Let E denote the Banach space of elements $\bar{u}$ under the maximum norm and $S = \{\bar{u} | \|\bar{u}\| \leq 1\}$. One can show that if f_1 is sufficiently large, then in a restricted time interval $A(S) \subset S$. This implies the existence of a solution. (Received November 16, 1961.)

587-106. M. BROWN, Institute for Advanced Study, Princeton, New Jersey, and B. CASSLER, University of Wisconsin, Madison, Wisconsin. On the representation of a manifold as a singular cell.

Theorem 1. Let M^n be a closed connected topological manifold. Then there is a map g of the n -cell I^n onto M such that $g|I^n$ is a homeomorphism, $\dim g(\dot{I}^n) \leq n-1$, $g(\dot{I}^n) \cap g(\dot{I}^n) = 0$. Theorem 2. Let M^n be a compact topological manifold with boundary B . Then there is a map g of $B \times [0,1]$ onto

M such that $g|_{B \times [0,1]}$ is a homeomorphism, $\dim g(B \times 1) \leq n - 1$, $g(B \times [0,1]) \cap g(B \times 1) = 0$, and for all $b \in B$, $g(b,0) = b$. Corollary. Let $R = g(B \times 1)$. Then M is homeomorphic to the mapping cylinder of a map from B onto R. (Received November 16, 1961.)

587-107. G. W. DAY, Purdue University, West Lafayette, Indiana. Super-atomic Boolean algebras. Part II. Preliminary report.

(See Abstracts 581-30 and 61T-266, Notices Amer. Math. Soc. vol. 8 (1961) pp. 255 and 518.) This paper considers the following question: What Boolean algebras have free complete extensions? H. Gaifman recently proved (Abstract 586-9, Notices Amer. Math. Soc. vol. 8 (1961) p. 510) that no infinite free Boolean algebra has a free complete extension, thus solving a problem posed by L. Rieger. Using results from the abstracts cited above and this result of Gaifman, one is able to prove the following result: A Boolean algebra has a free complete extension if and only if it is super-atomic. (Received November 16, 1961.)

587-108. LARRY ARMIJO, 3911 Mt. Vernon, Houston, Texas. An integral representation theorem for functions convex with respect to a pair of functions.

Let $\phi_1(x)$ and $\phi_2(x)$ be continuous in an open interval (a,b) and let $K(x,t) = \phi_1(t)\phi_2(x) - \phi_2(t)\phi_1(x) > 0$ for $a < t < x < b$. A function $y(x)$ defined on (a,b) is said to be convex with respect to ϕ_1 and ϕ_2 on (a,b) if and only if $D(\phi_1(x_j), \phi_2(x_j), y(x_j)) \geq 0$ for arbitrary $a < x_1 \leq x_2 \leq x_3 < b$, where $D(\phi_1(x_j), \phi_2(x_j), y(x_j))$ is the 3 by 3 determinant whose j th row is $\phi_1(x_j), \phi_2(x_j), y(x_j)$. The convex function which has been defined is easily seen to be "sub-F" in the sense defined by E. F. Beckenbach. Imposing certain differentiability properties on ϕ_1, ϕ_2 and y , results are obtained which correspond to certain well known results for ordinary convex functions. For example, if $\phi_1, \phi_2, y \in C^2$, and if the Wronskian of ϕ_1 and ϕ_2 is strictly positive throughout (a,b) , then y is convex with respect to ϕ_1 and ϕ_2 if and only if the determinant $D(\phi_1^{(j-1)}(x), \phi_2^{(j-1)}(x), y^{(j-1)}(x)) \geq 0$. Dropping the differentiability requirements, it has been shown that y is convex with respect to ϕ_1 and ϕ_2 if and only if there exists a monotone increasing function $G(x)$ and constants A, B , such that $y(x) = \int_{x_0}^x K(x,t)dG(t) + A\phi_1(x) + B\phi_2(x)$, where $x_0 \in (a,b)$. The function $G(t)$ is unique to within an additive constant except possibly at a countable number of points. (Received November 16, 1961.)

587-109. SIBE MARDESIC, University of Zagreb, Zagreb, Yugoslavia, and JACK SEGAL, University of Washington, Seattle 5, Washington. ϵ -mappings onto manifolds. Preliminary report.

An ϵ -mapping of a metric space A onto B is a continuous function f of A onto B such that for each b in B, the set $f^{-1}(b)$ has diameter less than ϵ . Theorem 1. Let X be a metric continuum and let $\mathcal{P} = \mathcal{P}_\alpha$ be a family of polyhedra such that for every $\epsilon > 0$ there exists an ϵ -mapping of X onto some $P_\alpha \in \mathcal{P}$. Then X is the inverse limit of some inverse sequence $\{P_i; \pi_{ij}\}$, $i, j = 1, 2, \dots$, where all $P_i \in \mathcal{P}$ and each bonding map π_{ij} maps P_j onto P_i . A space A is said to be locally cyclic provided, for any $x \in A$ and neighborhood U of x, there is a neighborhood (open) V of x, $V \subset U$, which has no cut points. Theorem 2. Let A be a locally cyclic continuum such that, for every $\epsilon > 0$, A admits ϵ -mappings onto B an orientable 2-dimensional manifold (with or without boundary). Then A is

homeomorphic to B. For the 2-dimensional case this theorem contains results of Borsuk (there do not exist ϵ -mappings of the 2-sphere onto the disc for all $\epsilon > 0$), Kuratowski (there do not exist ϵ -mappings of the disc onto the 2-sphere for all $\epsilon > 0$) and Fort (there do not exist ϵ -mappings of the disc onto the torus for all $\epsilon > 0$). It also partially answers a question of Kuratowski and Ulam (Scottish Book, Problem 97). (Received November 14, 1961.)

587-110. COSTAS KASSIMATIS, Box 5548, State College Station, Raleigh, North Carolina.
Zero-sets in topological spaces.

The present paper investigates the relations between the families of the zero-sets in topological spaces. (Notation and terminology are as in Gillman-Jerison, Rings of continuous functions.) Theorem. Let X and Y be topological spaces and let Y be completely regular. Assume that every function $f \in C(X)$ has a factorization $g \circ h$, where $g \in C(Y)$ and h is a mapping from X into Y such that $t \in C(Y)$ implies $t \circ h \in C(X)$. Then, $Z(X)$ coincides with the family of sets $s_i^+ [Z(Y)]$ for all $s_i \in C(X, Y)$. If the space Y is identified with the space of real numbers R then the following known result is obtained as a Corollary: Let X be a topological space. A base for the closed sets in the weak topology induced by $C(X)$ on X is the family $Z(X)$ of all zero-sets. (Received November 14, 1961.)

587-111. J. P. CLAY, 5318 Baynton Street, Philadelphia 44, Pennsylvania. Proximity relations in transformation groups.

Let (X, T) be a transformation group with compact phase space X and arbitrary phase group T . As a general reference, consult the colloquium volume of Gottschalk and Hedlund (Topological dynamics). P is the previously known proximal relation; Q , the regionally proximal relation. Let $(x, y) \in X \times X$. $(x, y) \in L$ if for each index α of X there exists a syndetic subset A of T such that $(xt, yt) \in \alpha$ for all $t \in A$. $(x, y) \in M$ if for each index α and neighborhoods U of x and V of y , there exists a syndetic sub-set A of T and $x_1 \in U$, $y_1 \in V$ such that $(x_1 t, y_1 t) \in \alpha$ for all $t \in A$. It is shown that when P is closed, $P = L$ and P is an invariant closed equivalence relation. L is characterized as the union of all orbit closures under $(X \times X, T)$ contained in P ; it is shown that $P = Q$ implies that (X, T) is coterminous, i.e., $P = Q = L = M$; and the productivity of L and M are proven. There are several results which describe L , M , P , and Q under such hypotheses as X minimal, T abelian; T regionally mixing; and the like. Finally, the author has weakened substantially the hypothesis of a theorem recently proven by J. D. Baum (An equicontinuity condition for transformation groups, Proc. Amer. Math. Soc. vol. 12 (1961) pp. 30-32). (Received November 14, 1961.)

587-112. A. T. BUTSON, Box 8052, University of Miami, Coral Gables 46, Florida. Relative difference sets.

Let $\mathcal{G} = \langle G, \cdot \rangle$ be a group of order v , N a subset of G containing n elements including 1, and $K = G - N$. A subset D of k elements of G is called a difference set of G relative to N if the following holds: (1) If $x \in K$, there are exactly δ distinct ordered pairs (d_1, d_2) of distinct elements of D such that $x = d_1^{-1} d_2$; and exactly δ distinct ordered pairs (d_3, d_4) of distinct elements of D such that $x = d_3 d_4^{-1}$. (2) Both $d_5^{-1} d_6$ and $d_5 d_6^{-1}$ are in K for any pair (d_5, d_6) of distinct elements of D . In this

paper analogues of some of the well-known results concerning ordinary difference sets ($N = \{1\}$) are obtained. (Received November 14, 1961.)

587-113. E. D. NERING, Arizona State University, Tempe, Arizona. Coalition bargaining in N-person games.

In an n-person game in which the only strategic maneuvers are to form coalitions it is assumed that coalitions can be built up only in successive steps in which two smaller coalitions unite to form a larger coalition. Once a larger coalition is formed it cannot be dissolved so that all possible splits of future gains must be anticipated in a contract at the time the coalition is formed. If A and B are the coalitions which unite and $u(A, B)$ is a future payoff to be split and x_A is the share A expects for the strategic situation regardless of its partner, then $x_A + x_B \geq u(A, B)$. For each strategic situation a system of inequalities of this type is obtained. An effective solution is one in which the inequalities satisfied as equalities are of rank equal to the number of coalitions competing. Consideration of these systems of inequalities for each strategic situation determines the reasonable shares for each party to a contract. (Received November 17, 1961.)

587-114. M. N. BLEICHER, University of California, Berkeley 4, California. Lattice coverings of n-dimensional euclidean space by spheres.

Let Λ be a lattice (i.e. a group generated by n independent vectors $X_1, X_2, \dots, X_n$) in E_n . Let r be the least real number such that every point in E_n is not further than r from some point of Λ . The density of the covering of E_n by spheres which corresponds to Λ is denoted by $\theta_n(\Lambda)$ and defined by $\theta_n(\Lambda) = r^n J_n / d(\Lambda)$, where J_n is the volume of the unit sphere in E_n and $d(\Lambda)$ is the determinant of Λ (i.e. the absolute value of the determinant in which the ith row is X_i). The problem is to find which lattices best cover E_n by spheres, i.e. minimize $\theta_n(\Lambda)$. For each n, the author constructs a lattice Λ_n in E_n such that: (i) $\min\{\theta_n(\Lambda)\} \leq \theta_n(\Lambda_n) = (1/\Gamma((n+2)/2)) [\pi n(n+2)/12(n+1)^{1-1/n}]^{n/2}$, (ii) each of the lattices Λ_n yields a relative minimum of $\theta_n(\Lambda)$, and (iii) for large n, $\theta_n(\Lambda_n)$ is not the absolute minimum of $\theta_n(\Lambda)$. The lattice Λ_4 has been conjectured by R. P. Bambah to be best possible; thus (ii) strengthens this conjecture. The proof is accomplished by restating the problems in terms of quadratic forms and applying the reduction by parallelohedra of G. Voronoi. (Received November 17, 1961.)

587-115. H. D. BRUNK, University of California, Riverside, California. On regression, projection, and conditional expectation. Preliminary report.

Often a regression problem may be viewed as the problem of finding the point of a convex cone and lattice in the space, L_2 , of square-integrable random variables, which is nearest a given point. Let Ω be a space of "elementary events", $\mathcal{A}$ a σ -field of subsets of Ω , and μ a measure on $\mathcal{A}$. In line with recent suggestions, μ is not assumed normed, or even necessarily σ -finite. A theorem yielding projection on a closed convex cone and lattice, $\mathcal{C}$, in L_2 is extended so as to associate with each element f of L_1 a random variable g such that $\int gh d\mu \geq \int fh d\mu$ for all bounded h in $\mathcal{C}$. When

$\mathcal{C}$ is the class of functions in L_2 measurable with respect to a sub- σ -field of $\mathcal{A}$, g is the conditional expectation of f given $\mathcal{C}$. (Received November 17, 1961.)

587-116. D. B. COLEMAN, Box 1514, Vanderbilt University, Nashville 5, Tennessee.

A theorem on finite group algebras.

Let G be a finite group and F a field whose characteristic does not divide the order of G . Let $G(F)$ denote the group algebra of G over F . Theorem. Let G be the direct product of groups G_i , H the direct product of groups H_i , $i = 1, 2, \dots, n$, with $|G_i| = |H_i| = n_i$ and $(n_i, n_j) = 1$ if $i \neq j$. Then $G(F) \cong H(F)$ if and only if $G_i(F) \cong H_i(F)$ for each $i = 1, 2, \dots, n$. Corollary. Two finite nilpotent groups have isomorphic group algebras if and only if their corresponding Sylow subgroups have isomorphic group algebras. This theorem was proved for Abelian groups by Perlis and Walker in their paper Abelian group algebras of finite order, Trans. Amer. Math. Soc. vol. 68 (1950). (Received November 17, 1961.)

587-117. E. GAGLIARDO, University of Kansas, Lawrence, Kansas. Interpolation spaces.

Let A_1, A_2, B_1, B_2 be Banach spaces with $A_1 \subset A_2, B_1 \subset B_2$ (the imbeddings being linear and continuous). Denote by $\mathcal{J}(A_1, A_2, B_1, B_2)$ the class of quasi-linear transformations T , bounded with respect to the spaces A_i, B_i , i.e. such that: (1) $T(A_1) \subset B_1$ and $\|Tu\|_{B_1} \leq c\|u\|_{A_1}$, for $u \in A_1$; (2) for $v_i \in A_i, T(v_1 + v_2) = T_{v_2}v_1 + T_{v_1}v_2$ where $T_z u$ satisfies (1) with c independent of z . A Banach space C such that $A_1 \subset C \subset A_2$ is called an intermediate space between A_1 and A_2 . A quasi-linear interpolation space is an intermediate space which is invariant under transformations $T \in \mathcal{J}(A_1, A_2, B_1, B_2)$. Not every intermediate space is an interpolation space. All the quasi-linear interpolation spaces (and only such) can be constructed in the following way: Let $u \in A_2$; consider the set $M(u)$ of points (x, y) such that there exist $v_1 \in A_1, v_2 \in A_2$, with $v_1 + v_2 = u, \|v_1\|_{A_1} \leq x, \|v_2\|_{A_2} \leq y$. If $F[M]$ is a functional satisfying certain hypotheses one can define the interpolation space $F(A, B)$ by $F[M(u)] < +\infty$. Several compactness and interpolation theorems can be derived. By means of the functional $F[M] = (\int_{\partial M} x^\alpha y^\beta |y'|^\gamma dx)^{1/(\alpha+\beta+1)}$ it is possible to interpolate between the L^p spaces, and also the spaces of potentials P^r . (Received November 17, 1961.)

587-118. G. W. HEDSTROM, West Engineering, University of Michigan, Ann Arbor, Michigan, and JACOB KOREVAAR, Stanford University, Stanford, California. The zeros of the partial sums of certain small entire functions.

In order to shed some light on the location of the zeros of the partial sums of the power series of entire functions of order zero, the polynomials $s_n(z) = \sum_{p=0}^n \exp(-p^k) z^p, 1 < k < 2$, are studied. The main tool is the use of Poisson's transformation on $g(z) = \sum_{p=-\infty}^{\infty} \exp(-|p|^k) z^p$. The changes of sign of $g(-x)$ are used to show that all but at most $O(n^{2-k})$ of the zeros of the s_n are negative real. It is also shown that for $n > n_0(\delta, \epsilon)$ the zeros of s_n in $|\arg z| \leq \pi - \epsilon, |z| \leq r_0(\epsilon)$, satisfy the inequalities $(1 - \delta) \exp(kn^{k-1}) \leq |z| \leq (1 + \delta) \exp(kn^{k-1})$. Finally, the Boutroux-Cartan lemma is used to prove that for every angle Ω with vertex 0 there are constants $c > 0$ and n_1 such that for $n > n_1$ the number of zeros of s_n in Ω is greater than cn^{2-k} . (Received November 17, 1961.)

587-119. R. V. KADISON, Box 314, Hamilton Hall, Columbia University, New York 27, New York. Normalcy in operator algebras.

A von Neumann (v.N.) subalgebra R_0 of a v.N. algebra R is normal in R when it enjoys the double commutant property relative to R (i.e., $(R_0 \cap R)' \cap R = R_0$). With R all bounded operators normalcy of R_0 is von Neumann's double commutant theorem. Examples where normalcy fails to hold in general were given by Murray and von Neumann when R is a factor of type II or III. We show that if R_0 is of type I it is normal in R if and only if the center C_0 of R_0 is normal in R . With A an abelian von Neumann subalgebra of R , A is normal in R if and only if: (a) A is the intersection of all maximal abelian subalgebras of R containing it; (b) The von Neumann algebra generated by A and R' has A as center; (c) In the separable case, the direct integral reduction of R' relative to A yields factor representations almost everywhere. If A is "discrete over the center C of R ", A is normal in R . An example of an abelian von Neumann subalgebra of a factor of type II₁ which is not normal in the factor is given. (Received November 17, 1961.)

587-120. SEYMOUR SHERMAN, Wayne State University, Detroit 2, Michigan. An analogue to the Witt identity.

The Witt identity may be formulated in terms of a free semigroup with generators $a_1, \dots, a_k$. Let $M(m_1, \dots, m_k)$ be the number of primitive (i.e., same length and period), circular words with m_i occurrences of a_i , $1 \leq i \leq k$. Let $z_1, \dots, z_k$ be commuting indeterminates. Then $1 - z_1 - \dots - z_k = \prod_{m_1, \dots, m_k \geq 0} (1 - z_1^{m_1} \dots z_k^{m_k})^{M(m_1, \dots, m_k)}$. The result may be interpreted in terms of one-way (e.g., counterclockwise) circular nonperiodic paths on a set of k loops all of which are in a plane, intersect at a single point P , and no two of which have any other point in common. Further, no two of the loops have a common tangent at P . An analogue to the above identity is presented for a free group with k generators $b_1, \dots, b_k$, where now the binomial factors on the right are sums or differences depending on the oddness or evenness of the algebraic number of rotations of the tangent vector on the circular, nonperiodic, two-way path corresponding to the primitive, circular, reduced word in the group, where the analogue to m_i is equal to the number of appearances of b_i plus the number of appearances of b_i^{-1} in the word and where now the left-hand side of the identity is $((1 + z_1)(1 + z_2) \dots (1 + z_k))^2$. (Received November 17, 1961.)

587-121. A. B. SIMON, Northwestern University, Evanston, Illinois. The Šilov boundary of a subalgebra of measures on a group.

Let G be a locally compact Abelian group and let $M(G)$ be the algebra of all bounded regular Borel measures on G . Suppose P is an independent subset of G . Then generate the subalgebra M (as in Abstract 576-42, Notices Amer. Math. Soc. vol. 7 (1960) p. 902). Theorem. The Šilov boundary of M is its entire ideal space. (Received November 17, 1961.)

587-122. MARVIN SHINBROT, University of Chicago, Chicago 37, Illinois. Difference kernels.

Let $K(w)$, the Fourier transform of the real function $k(|x|)$, satisfy the following two conditions: (i) $(d/dw)[\log |K(w)|]$ is continuous for all real w and is in L^2 ; (ii) as $w \rightarrow \infty$ along the real axis, the quantities $\lambda = \limsup \log |1/K(w)|/\log w$ and $\mu = \liminf \log |1/K(w)|/\log w$ are finite. Then, it is proved that the integral equation $g(x) = \int_{-a}^a k(|x-t|)f(t)dt$ ($-a < x < a$) has a unique solution for all $g \in C^2$ whenever a is large enough and λ is less than unity. The solution is of class $L^{2/\lambda-\epsilon}$. If $\lambda \geq 1$, the solution still exists (when a is large enough), but it is then a distribution whose Fourier transform is $O(|w|^{\lambda/2-1+\epsilon})$. Finally, the case of complex k is also analyzed, but an additional condition is required. (Received November 17, 1961.)

587-123. ABOULGHASSEM ZIRAKZADEH, University of Colorado, Boulder, Colorado. Graphical solution of problems involving the incidence of points, lines and planes.

A transformation T which transforms the Euclidean n -space E_n into another space P_n , which lies in an Euclidean plane, is introduced. It is shown that the incidence properties of P_n are identical with those of E_n , and thus it is possible to study the space E_n by studying the space P_n . In particular some problems involving incidence of points, lines and planes can be solved graphically. The following two problems are discussed and solved by this method: (1) In a 3-space, draw a line t through a given point A and intersecting two given lines t_1 and t_2 . (2) Solve 4 linear equations in 4 unknowns, graphically. (Received November 17, 1961.)

587-124. MAX KOECHER, Institute of Technology, University of Minnesota, Minneapolis, Minnesota. Jordan-algebras and their applications to analysis.

We consider pairs (Y, w) where Y is a domain in some real vector space X and $w = w(y)$ a real analytic function in Y fulfilling certain properties. $w(y)$ introduces a Riemannian metric in Y . The geodesics of this metric through a given point c can be put in form $y(\tau) = e(\tau x)$; $e(x) = \sum_{n=0}^{\infty} (1/n!)x^n$ denotes here the exponential of a suitable semi-simple Jordan-algebra in X . Conversely a semi-simple Jordan-algebra gives rise to a pair (Y, w) with the above mentioned properties. Therefore there is a bijective correspondence between these pairs (Y, w) and the semi-simple real Jordan-algebras. (Received November 17, 1961.)

587-125. JAMES VAN BUSKIRK, University of Wisconsin, Madison 6, Wisconsin. On the braid groups of the projective plane.

Braid groups of arbitrary manifolds have been defined by R. H. Fox using a modification of E. Artin's original idea of braids of the plane (see Bull. Amer. Math. Soc. vol. 67 (1961) pp. 211-213). If M is a manifold, let $B_n(M)$ denote the braid group of M on n strings. A result due originally to L. Neuwirth states that a necessary condition that a compact 2-manifold M have braid groups containing elements of finite order is that M be the 2-sphere S^2 or the projective plane P^2 (Math. Scand. vol. 9 (1961), in press). E. Fadell and the author have shown that $B_n(S^2)$ has elements of finite order for every n . The objective of this paper is to give a presentation of $B_n(P^2)$ and thus obtain the theorem: A necessary and sufficient condition that $B_n(M)$ contain elements of finite order, where M

is a compact 2-manifold, is that M be S^2 or P^2 . The proof uses results of E. Fadell (On homotopy groups of configuration spaces and the string problem of Dirac, to appear in Duke Math. J.) and techniques employed previously in computing $B_n(S^2)$ by fiber space methods. (Received November 17, 1961.)

587-126. RUSSELL REMAGE, JR., University of Delaware, Newark, Delaware. An extension of a fine-cyclic element additivity theorem.

Let S be a cyclic Peano space. A C-set is a continuum of S whose boundary is a finite set. An irreducible C-chain C between points a and b of S is a C-set in which there is an arc α joining a and b such that each fine-cyclic element of S contained in C meets α in a connected set, and such that if F and F' are fine-cyclic elements contained in C with nonempty intersection, this intersection meets α . S is said to have an approximation by C-chains if every fine-cyclic element is a C-set, and if, for each C-set, there is an irreducible C-chain joining each pair of points of the boundary of C . C. J. Neugebauer (Illinois J. Math. vol. 2 (1958) pp. 396-401) proved a fine-cyclic element additivity theorem for a class of functionals on the mappings of a Peano space to a Peano space which have factorizations whose middle space is of finite degree of multicoherence. This result is extended by replacing the requirement that the middle space be of finite degree of multicoherence by the condition that each cyclic element of the middle space has an approximation by C-chains. (Received November 17, 1961.)

587-127. B. J. BALL, University of Georgia, Athens Georgia, and W. R. ALFORD, Tulane University, New Orleans, Louisiana. Some almost polyhedral wild arcs.

An arc in E^3 which is locally polyhedral except at one of its end points will be said to be almost polyhedral. The penetration index, $P(A, x)$, of an arc A at a point x of A is defined to be the smallest cardinal number n such that there are arbitrarily small 2-spheres enclosing x and containing no more than n points of A . The purpose of this paper is to generalize the construction of Ralph H. Fox and Emil Artin (Example 1.1 of Some wild cells and spheres in three dimensional space, Ann. of Math. vol. 49 (1948) pp. 979-990) to obtain a collection of almost polyhedral arcs $\{A_n, n = 1, 2, \dots\}$ such that $P(A_n, x_n) = 2n + 1$, where x_n is the bad point of A_n . Since the penetration index at a point x of an arc is an embedding invariant, no two of the A_n 's are equivalently embedded. (Received November 17, 1961.)

587-128. STANLEY TENNENBAUM, Logic of Computers Group, 4001 Angell Hall, University of Michigan, Ann Arbor, Michigan. Degree of unsolvability and the rate of growth of functions.

Call the strictly increasing sequence of positive integers whose range is A the principal function (p.f.) of A . It is known that (i) there exist recursively enumerable (r.e.) sets whose complements, though infinite, are so sparse that their p.f.s are not dominated by any recursive function and that (ii) every r.e. degree > 0 is represented by a r.e. set whose complement is sparse in this sense. Since any h which dominates every partial recursive function has degree $\geq 0'$, the question arises whether there is a r.e. set T whose complement T' is so sparse that it has such an h as its p.f. A

positive answer follows from the Lemma. To every partial recursive sequence of functions $q_n(x)$ there effectively corresponds a r.e. set S such that the p.f. h of S' has the property that for each n , $h(x) > q_n(x)$ for $x > n$ whenever $q_n(x)$ is defined. But the T' obtained is not indecomposable (ind). A is ind, if A is infinite, yet for no r.e. set F are $A \cap F$ and $A \cap F'$ both infinite. R. M. Friedberg has constructed r.e. sets with ind. complements (maximal simple sets). These sets lie at the end of a progression of types of r.e. sets studied by E. Post in a scheme for discovering unsolvable r.e. sets of low degree by constraining their complements to be ever more sparse. However it can be shown that every maximal simple set has degree $0'$. (Received November 17, 1961.)

587-129. F. B. KNIGHT, 119 Folwell Hall, Minneapolis 14, Minnesota. An entrance boundary for Markov processes.

Let $X(t)$ be a Markov process (random function) on $(\Omega, \mathcal{F}, P)$ relative to fields $\mathcal{F}(t)$, with measurable range space $(X, \mathcal{B})$, and with transition function $p(t, x, E)$ measurable in (t, x) . Suppose that $p(., x_1, .) = p(., x_2, .)$ if and only if $x_1 = x_2$, that wide-sense conditional distributions over $\mathcal{B}$ exist, and that $\{E_i\}$ is a countable algebra of sets generating $\mathcal{B}$. For $\lambda > 0$, $g(i) > 0$, $\sum_i g(i) < \infty$, define a metric $\rho_\lambda(x, y)$ on X by setting $\rho_\lambda(x, y) = \sum_i g(i) |f_i(x) - f_i(y)|$ where $f_i(x)$ is $\int_0^\infty e^{-\lambda t} p(t, x, E_i) dt$ divided by $\sum_i g(i) \int_0^\infty e^{-\lambda t} p(t, x, E_i) dt$, and let $(Y_\rho, \mathcal{B}_\rho)$ be the completion of X in the metric ρ_λ and the topological σ -algebra. Define a process $Y_\rho(t)$ with range Y_ρ by $Y_\rho(t) = \lim_{\tau \uparrow t, \tau \text{ rational}} X(\tau)$ when the limit exists for all t , and $Y_\rho(t) = \underline{x}$ otherwise, where $\underline{x}$ is a fixed element of X . Theorem. $Y_\rho(t)$ is a strong Markov process with range $(Y_\rho, \mathcal{B}_\rho)$, with paths continuous from the right, and, except for t in some countable set, $P\{Y_\rho(t) = X(t)\} = 1$. (Received November 21, 1961.)

587-130. AUBERT DAIGNEAULT, University of Montreal, Montreal, P. Q., Canada. Extension of isomorphisms of polyadic algebras.

Let "algebra" mean "locally finite I- polyadic algebra of infinite degree with equality". If A is an algebra and L is a set of constants of an extension of A , $A(L)$ denotes the algebra generated by A and L . An extension $A(K)$ obtained by fixing the variables of $K = I^+ - I$ in an $I^+ -$ dilation of A is called a free extension of obvious reasons. Let X and I be infinite sets, let C be the algebra of all finite dimensional functions from X^I to $\{0, 1\}$ and let $K = \{k\}$. Theorem 1. The extension $C(k)$ has a set of constants L such that $\overline{L} = \overline{X}$ and $C(L)$ is a free extension. Theorem 2. Let A_1 and A_2 be subalgebras of a simple algebra A and let $\sigma: A_1 \rightarrow A_2$ be an epimorphism. Then there exists a simple rich extension B of A such that, if Y is the set of all constants of A , σ extends to an epimorphism $A_1(Y) \rightarrow A_2(Y)$. Corollary 1. For any simple algebra A there exists a simple rich extension B such that whenever A_1 and A_2 are subalgebras of A and $\sigma: A_1 \rightarrow A_2$ is an epimorphism, σ extends to an automorphism of B . Corollary 2. Let A_1 be a subalgebra of a simple algebra A_2 . Then there exists a simple rich extension B of A_2 and an automorphism of B that leaves A_1 elementwise fixed and moves every element of $A_2 \setminus A_1$. From Corollary 2 it is easy to derive Beth's theorem and a theorem of Svenonius. (Theoria 1960). (Received August 15, 1961.)

ABSTRACTS PRESENTED BY TITLE

61T-272. J. H. EATON and B. H. WHALEN, Department of Electrical Engineering, University of California, Berkeley, California. A fixed point theorem with an application to the control of finite state stochastic systems.

Let X be an n -dimensional vector space over the reals, with some norm, $\|\cdot\|$, and with the vector $(x_1, \dots, x_n)$ denoted by x . Let there be a family of transformations of X into itself defined by $y = T(\pi)x = A(\pi)x + c(\pi)$ where $A(\pi)$ is a matrix, π and $c(\pi)$ are vectors, and y_i depends only on π_i . Each π_i is a member of a finite parameter set Π_i and π may be any member of their cartesian product, Π . Define the partial ordering $\leq$ on X by $x \leq y \iff x_i \leq y_i$ for all $i \leq n$. Let the $A(\pi)$ be non-negative transformations such that (i) there exists a uniformly positive vector, a , such that $A(\pi)a \leq a$ for all π in Π , (ii) no $T(\pi)$ has a fixed point with a component of $-\infty$, (iii) $\|A(\pi)\| \leq 1$ for all π in Π and $\|A(\pi)^k\| < 1$ for some π and some k . It is shown that the mapping $Mx = \min\{y: y = T(\pi)x, \pi \in \Pi\}$ has a unique fixed point x^0 ; that $x^0 = \min\{x: x = T(\pi)x, \pi \in \Pi\}$; that, for all x , $M^n x \rightarrow x^0$ as $n \rightarrow \infty$; and that, if x is a fixed point of some $T(\pi)$, $\pi \in \Pi$, the convergence is monotone, whereas, if x is any point $\geq x^0$ the norm of the difference $(M^n x - x^0)$ goes to zero monotonically. This result is applicable to the determination of optimal control policies in finite state stochastic systems. (Received September 29, 1961.)

61T-273. MARSTON MORSE, Institute for Advanced Study, Princeton, New Jersey. Differentiable isotopies.

In a differentiable isotopy, as defined by Milnor, the intermediate diffeomorphisms F_t , $-\infty < t < \infty$, map a differentiable $(n-1)$ -manifold M_1 in a euclidean n -space E onto a fixed manifold M_2 in E . In this paper M_2 is permitted to vary with t in E . Let S be an $(n-1)$ -sphere in E . For simplicity diffeomorphisms are restricted to those of class C^∞ . A necessary and sufficient condition that a diffeomorphism f of S into E be extendible as a sense-preserving diffeomorphism over E is that f be differentiably isotopic to the identity in the sense meant here. This paper also shows that diffeomorphisms of the "type of the identity" are "everywhere dense" among "sense-preserving" diffeomorphisms of S into E . (Received September 28, 1961.)

61T-274. R. S. FREEMAN, University of California, Radiation Laboratory, P.O. Box 808, Livermore, California. Closed extensions of uniformly elliptic, second order differential operators determined by a general class of boundary conditions.

Let $\mathcal{D}$ be a bounded domain in E^m ($m > 1$) with $\partial\mathcal{D}$ an $m-1$ dimensional C^2 manifold. Let $A = -\sum D_j a_{ij} D_i + c$ be a formally self-adjoint, uniformly elliptic operator with coefficients in $C^2(\mathcal{D})$. Let $\mathcal{D}_1(\mathcal{D}) = \{u \in W^{1,2}(\mathcal{D}): (i) u \in W_{loc}^{2,2}(\mathcal{D}), (ii) Au \in \mathcal{L}^2(\mathcal{D}), (iii) u, u_\nu \text{ have } \mathcal{L}^2 \text{ boundary values}\}$ where $u_\nu = \sum_{j=1}^m a_{ij} D_i u \cos(u, x_j)$, u being the unit normal to $\partial\mathcal{D}$. Let L be a bounded operator on $\mathcal{L}^2(\partial\mathcal{D})$ and define $\mathcal{D}(T_L) = \{u \in \mathcal{D}_1(\mathcal{D}): \tilde{u}_\nu - L\tilde{u} = 0\}$, $\tilde{u}$ denoting the $\mathcal{L}^2$ boundary value of u . For

$u \in \mathcal{D}(T_L)$ let $T_L u = Au$. Then: Theorem. (i) T_L^1 is a closed linear operator in $\mathcal{L}^2(\mathcal{S})$, (ii) $(T_L)^* = T_L^*$, (iii) T_L has compact resolvent, (iv) $\sigma(T_L)$ lies in a parabolic region opening to the right. If $\mathcal{S}$ is an exterior region then (i), (ii), and (iv) are still true. If $\mathcal{S}$ and $\partial\mathcal{S}$ are unbounded, then (i) and (ii) are true if L is multiplication by a bounded, measurable function. These results extend those of R. S. Freeman (Abstracts 549-37 and 549-38) and W. G. Bade (Abstract 553-134), these Notices, vol. 5 (1958). (Received October 2, 1961.)

61T-275. W. D. L. APPLING, Duke University, Durham, North Carolina. Interval functions and the Hellinger integral. Preliminary report.

In this paper all integrals discussed are Hellinger type limits of the appropriate sums. Suppose $[a, b]$ is a number interval, m is a real-valued nondecreasing function on $[a, b]$, and each of H and K is a real-valued bounded function of subintervals of $[a, b]$. Theorem 1. If for some number R , $0 \leq H[p, q] \leq R[m(q) - m(p)]$ for each subinterval $[p, q]$ of $[a, b]$, and $\int_{[a, b]} [H(I)dm]^{1/2}$ exists, then each of $\int_{[a, b]} \int_I [H(J)dm]^{1/2} / dm$ and $\int_{[a, b]} H(I)$ exists and equality holds. Theorem 2. If H is non-negative valued and $\int_{[a, b]} H(I)dm$ exists, then $\int_{[a, b]} [H(I)]^2 dm$ exists. Theorem 3. If each of $\int_{[a, b]} H(I)dm$ and $\int_{[a, b]} K(I)dm$ exists, then $\int_{[a, b]} H(I)K(I)dm$ exists. Theorem 4. If g is a real-valued function on $[a, b]$ having bounded variation on $[a, b]$, then $\int_{[a, b]} H(I)dg$ exists if and only if $\int_{[a, b]} H(I) \int_I |dg|$ exists. Theorem 5. If there is a number S such that $0 < S \leq |H[p, q]|$ for each subinterval $[p, q]$ of $[a, b]$ and $\int_{[a, b]} H(I)dm$ exists, then $\int_{[a, b]} [H(I)]^{-1} dm$ exists. (Received October 2, 1961.)

61T-276. A. S. FRAENKEL, University of Oregon, Eugene, Oregon. On a theorem of D. Ridout in the theory of diophantine approximations.

The following extension of the Thue-Siegel-Roth theorem was proved by Ridout (Mathematika 4, 1957). Let α be algebraic $\neq 0$. Let $S_1 = \{P_1, \dots, P_s\}$, $S_2 = \{Q_1, \dots, Q_t\}$ be sets of primes. Let μ, ν, c be real numbers satisfying $0 \leq \mu \leq 1$, $0 \leq \nu \leq 1$, $c > 0$. Let p, q be restricted to integers of the form (1) $p = p^* P_1^{\rho_1} \dots P_s^{\rho_s}$, $q = q^* Q_1^{\sigma_1} \dots Q_t^{\sigma_t}$, $\rho_1, \dots, \rho_s, \sigma_1, \dots, \sigma_t$ non-negative integers, p^*, q^* integers satisfying $|p^*| < c|p|^{\mu}$, $0 < q^* < cq^{\nu}$. Then (2) $0 < |\alpha - p/q| < q^{-(\mu+\nu+\zeta)}$ has only a finite number of solutions in p, q of the form (1) for every $\zeta > 0$. A proof is given based on Cassels' proof of the TSR theorem. It is shown that (2) has infinitely many solutions in p, q of the form (1) for almost no real α . For $\mu + \nu \geq 1$ and for $\mu = 0$, $0 \leq \nu \leq 1$ and $\nu = 0$, $0 \leq \mu \leq 1$, Ridout's Theorem is best possible, i.e., $|\alpha - p/q| < q^{-(\mu+\nu)}$ has infinitely many solutions in p, q of the form (1). If $S_1 \cap S_2 \neq \emptyset$, this holds also for $\mu + \nu < 1$. If $S_1 \cap S_2 = \emptyset$, $\mu + \nu < 1$ and $\delta > 0$, then $|\alpha - p/q| < q^{-(\mu+\nu-\delta)}$ has infinitely many solutions of the desired form for almost all real α . These results are specializations to the approximation of a single number, of theorems that are proved about simultaneous approximations. An upper bound to the number of solutions of (2) is given when α is algebraic. (Received October 2, 1961.)

61T-277. R. H. ROSEN, University of Michigan, Ann Arbor, Michigan. Stellar neighborhoods in polyhedral manifolds.

A subset of an n -manifold has been called cellular (by M. Brown) if it is the intersection of a

sequence of topological n -cells (C_i) so that C_{i+1} lies in the interior of C_i . Let Q be a subpolyhedron of a polyhedron P . Star Q is the union of all closed simplexes of P which meet Q . The stellar neighborhood of Q in P is Star Q minus the union of all closed simplexes of P which miss Q . If s is a simplex of P , s^* is the union of all closed simplexes of P which contain s . Now let M be an n -manifold with a triangulation T . The following theorems are proved. I. If K is a full subpolyhedron of M under T and K is cellular, then the stellar neighborhood of K is homeomorphic to E^n . II. If s is a simplex of T then the interior of s^* is homeomorphic to E^n . III. If t is a closed simplex in the first barycentric subdivision of T then t is cellular. These results are fairly easy consequences of the generalized Schoenflies theorem of Mazur and Brown and of the theorem below, which is actually derived from a stronger result. Theorem. Suppose X is a finite dimensional separable metric space and Y is the open cone over X from a point p . If K is a compact starlike set in Y then $Y - K$ is homeomorphic to $Y - p$. (A subset of Y is starlike if it contains p and each ray from p to X intersects it in a connected set.) (Received October 2, 1961.)

61T-278. O. L. MANGASARIAN, Shell Development Company, Emeryville, California.
Numerical solution of the first biharmonic problem by linear programming.

A numerical solution of the first biharmonic problem $\nabla^4 u = c$ in a rectangular region subject to $u = 0$ and $\partial u / \partial n = 0$ on the boundary is obtained by the well-known computational algorithm of linear programming. A novel feature of this method is that it is possible to give an upper bound on the error in the numerical solution in the interior of the region in terms of the objective function of the linear programming process. A monotonically decreasing curve is obtained which relates this bound to the number of terms in the numerical solution. While the present problem is of interest, this work may be considered as a successful test of a fairly general technique for solving linear boundary value problems. (Received October 2, 1961.)

61T-279. ALEXANDER WEINSTEIN, University of Maryland, Institute for Fluid Dynamics and Applied Mathematics, College Park, Maryland. The intermediate problems, Weyl's first fundamental lemma and the maximum-minimum theory of eigenvalues.

The basis of the maximum-minimum theory of eigenvalues is Weyl's fundamental lemma (Math. Ann. vol. 71 (1911) pp. 441-469; J. Reine Angew. Math. vol. 141 (1912) pp. 1-11). For the background and history of the problem see H. Weyl (Bull. Amer. Math. Soc. vol. 56, No. 2 (1950) pp. 124-128). It is shown in the present paper that a new proof can be obtained by the theory of intermediate problems. For a recent discussion of intermediate problems see, for example, A. Weinstein (Partial differential equations and continuum mechanics, University of Wisconsin Press, 1961, pp. 39-53). The new approach allows a more precise result concerning the conditions under which the maximum of the minimum is obtained. (Received October 2, 1961.)

61T-280. R. A. STRUBLE and S. M. YIONOULIS, North Carolina State College, Raleigh, North Carolina. General perturbational solution of the harmonically forced Duffing equation.

A general perturbational solution of the harmonically forced Duffing equation is obtained in the form of an asymptotic series. The general solution reveals the well-known characteristics of the harmonic, subharmonic and superharmonic responses. In addition, it reveals, perhaps for the first time, the global operation of the system and the intricate interplay of the system parameters. The long-period perturbations are exhibited in the form of amplitude and phase modulations for non-stationary oscillations while the short-period perturbations are exhibited as additive corrections to the principal motion. The periodic solutions (stationary oscillations) emerge in the absence of long-period perturbations and the asymptotic series then reduce to the Fourier expansions. (Received October 4, 1961.)

61T-281. A. A. MULLIN, University of Illinois, Urbana, Illinois. On some mutant-preserving transformations.

Let $(C, +, \cdot)$ be the field of complex numbers with θ as its additive identity. Let R be the mapping from the power set of $C - \theta$ into the power set of C defined by transforming elements of $C - \theta$ into their multiplicative inverses. Let S be the restriction of R to the power set of the set of all odd integers. E.g., Lemma. S carries an additive $(2, C)$ -mutant set to an additive $(2, C)$ -mutant set and any multiplicative mutant in the domain of R is carried by R to a multiplicative mutant set; for the definition of a (λ, T) -mutant set see Abstract 60T-34, Notices Amer. Math. Soc. vol. 7 (1960) pp. 1005-1006. Some less succinct theorems concerning well-known transformations, but in the context of mutant sets, are given. However a constructive exemplar for the existence of maximal mutants of $(C, +, \cdot)$ is needed to supplement the nonconstructive existence proof, given elsewhere, for maximal mutants in any arbitrary algebraic system. (Received October 9, 1961.)

61T-282. RONALD GRAHAM, 1863 La Loma Avenue, Apartment F, Berkeley 9, California. On sums of reciprocals of integers.

Let S be a sequence of positive integers and let $M(S)$ denote the monotone sequence $(\pi_1, \pi_2, \dots)$ of finite products of distinct elements of S . Suppose: (1) Every sufficiently large integer is the sum of distinct elements of $M(S)$. (2) There is a $c < 2$ such that $\pi_{n+1} \leq c \pi_n$ for all sufficiently large n . (3) $\pi_n \uparrow \infty$. The set of all numbers which are finite sums of reciprocals of distinct elements of $M(S)$ are characterized in this paper. Special cases of these results are: (1) r is the finite sum of reciprocals of distinct squares if and only if r is rational and $r \in [0, \pi^2/6 - 1) \cup [1, \pi^2/6)$. (2) If A, B, P, Q are positive integers, then P/Q is the finite sum of reciprocals of distinct terms of the arithmetic progression $Ak + B$, $k = 0, 1, 2, \dots$, if and only if $(Q/(Q, P(A, B)), A/(A, B)) = 1$ (where (m, n) denotes the g.c.d. of m and n). In particular, several questions raised by H. S. Wilf (Bull. Amer. Math. Soc. vol. 57 (1961) Research Problem 6, p. 456) can be answered, e.g., every arithmetic progression is an R -basis and there exist R -bases with zero density. (Received October 9, 1961.)

61T-283. N. A. WIEGMANN, 3801 Connecticut Avenue, Apartment 133, Washington 8, D. C.

Trace properties of semigroups of matrices with real quaternion elements.

In the theory of group representations by matrices with noncommutative elements the trace properties of matrices present difficulties which are not present in the commutative case (where trace properties are of great importance). The following direct analogs of theorems which are basic to the complex case are among results obtained here: If $\mathcal{A}$ and $\mathcal{B}$ are (finite or infinite) semigroups of matrices with real quaternion elements which are irreducible representations of a semigroup G , then $\mathcal{A}$ and $\mathcal{B}$ are (quaternion) similar if and only if the traces of all corresponding matrices are equal; if $\mathcal{A}$ and $\mathcal{B}$ are completely reducible representations of a semigroup G by $n \times n$ quaternion matrices, and if the traces of corresponding matrices are equal, then $\mathcal{A}$ and $\mathcal{B}$ are similar. (The first result generalizes a result obtained by J. Houle for the finite case.) In addition a complete analog of Burnside's Theorem is obtained. (Received October 10, 1961.)

61T-284. ANDREW BROWDER and JOHN WERMER, Brown University, Providence 12, Rhode

Island. Function algebras on an arc.

Let ψ be a real-valued continuous strictly decreasing function defined on $[0, \pi]$, with $\psi(0) = 0$, $\psi(\pi) = -\pi$. Let A be the algebra of all functions f , continuous on $[-\pi, \pi]$ and satisfying $f(t) = f(\psi(t))$, $t \in [0, \pi]$, and $\int_{-\pi}^{\pi} f(t)e^{int} dt = 0$, $n > 0$. One may regard A as a function algebra on $[0, \pi]$. If J is a Jordan arc in the complex plane, S the extended plane, there exists such a ψ , depending on J , such that A may be identified with the algebra of all functions continuous on S and analytic in $S - J$.

Theorem 1. A is a Dirichlet Algebra (i.e., the real parts of functions in A are uniformly dense in the space of all real continuous functions on $[0, \pi]$) if and only if $\psi'(t) = 0$ a.e. Theorem 2. If $\psi'(t) = 0$ a.e., the maximal ideal space of A is homeomorphic to the 2-sphere. Theorem 3. If $\psi'(t) = 0$ a.e., A is a maximal subalgebra of $C([0, \pi])$. Theorem 4. If $B = \{f \in A \mid f(0) = f(\pi)\}$, then when $\psi'(t) = 0$ a.e., B is a Dirichlet Algebra on the circle, not isomorphic to the usual one, or any subalgebra thereof. B is a maximal subalgebra on the circle. Theorem 5. There exists an arc J such that the induced ψ satisfies $\psi'(t) = 0$ a.e. (Received October 11, 1961.)

61T-285. WITHDRAWN.

61T-286. M. D. GEORGE, University of Missouri, Columbia, Missouri. On mixed problems for the nonlinear heat equation.

The one-dimensional nonlinear heat equation $(G(w))_t - (f(w)w_x)_x = h(x,t)$ in the rectangle R_T : $a < x < b$, $0 < t < T$ is rewritten as a first order system with new independent variables $u_1 = e^{-\lambda t}G(w)$, $u_2 = e^{-\lambda t}f(w)w_x$ (cf. Friedrichs, CPAM vol. 11 (1958)). In the Hilbert space $H = L^2(R_T) \times L^2(R_T)$, the system can be written in the form $(L + K_\lambda)u = h_\lambda$, where L is linear and K_λ is nonlinear but continuous in H . The functions f, G' are assumed Lipschitz continuous, bounded, and bounded away from zero. The boundary conditions imposed on the solution are quite general, being described in terms of a boundary matrix M as in the Friedrichs paper. Theorem. Let $z(x,t) \in L^2(R_T)$ be a strong solution of the heat equation satisfying homogeneous boundary conditions as described and suppose $f(z)z_x$ is essentially bounded. Then there is no other strong solution satisfying the boundary conditions. A strong solution is the L^2 limit of appropriate C^2 functions. The proof rests on certain inequalities in H . Other results concerning continuous L^2 dependence of the solution on $h(x,t)$ and the initial data are also obtained. (Received October 17, 1961.)

61T-287. F. J. ALMGREN, JR. Brown University, Providence, Rhode Island. The homotopy groups of the integral cycle groups.

For $B \subset A \subset \mathbb{R}^n$, let $Z_k(A, B)$ be the group of those k dimensional integral currents of $\mathbb{R}^n$ whose supports lie in A and the supports of whose boundaries lie in B , for $k = 1, 2, 3, \dots$ (see Federer and Fleming, Ann. of Math. vol. 72 (1960) pp. 458-520). Also let $Z_0(A, B)$ be the group of those 0 dimensional integral currents of $\mathbb{R}^n$, with support in A , whose coefficient sum is 0. For $T_1, T_2 \in Z_k(A, B)$, let $G(T_1, T_2) = \inf\{M(T_1 - T_2 - \partial S) + M(S) : S \in Z_{k+1}(A, A)\}$. G is a metric and topologizes $Z_k(A, B)$. Inductive limit topologies based on G in which compact sets are M bounded also exist. With each of these topologies we have: Theorem. For any two compact Lipschitz neighborhood retracts A and B in $\mathbb{R}^n$, with $B \subset A$, the homotopy group $\pi_i(Z_k(A, B); 0)$ is naturally isomorphic with the homology group $H_{i+k}(A, B)$; here $i, k = 0, 1, 2, \dots$. Similar results hold for the relative groups of pairs $(Z_k(A, \emptyset), Z_k(B, \emptyset))$ and with related topologies for $Z_k(A, B)/Z_k(B, B)$. For $k = 0$ we have essentially the theorem of Dold and Thom on symmetric product spaces, but the methods are entirely different. (Received October 17, 1961.)

61T-288. ECKFORD COHEN, University of Tennessee, Knoxville 16, Tennessee. Some analogues of certain arithmetical functions.

Let r and n denote positive integers and let $\phi(n)$ be the Euler totient. Let $a_r(n)$ denote the maximal unitary $(r+1)$ -free divisor of n and place $\bar{\phi}_r(n) = (n/a_r(n))\phi(a_r(n))$. The function $\phi_r(n)$ is evaluated in terms of a function $\mu_r(n)$ corresponding to the Möbius function. Using properties of $\mu_r(n)$, the average order of $\phi_r(n)$ is determined. In particular, it is shown that $\sum_{n \leq x} \bar{\phi}_r(n) = a_r x^{2/2} + O(x \log^2 x)$ where $a_r = \prod (1 - 1/p^2 + 1/p^{r+2})$, the product ranging over the primes p . A similar estimate for an analogue of the characteristic function of the k -free integers is deduced. The method used involves only well-known properties of (real) Dirichlet series. (In Abstract 575-34, Notices Amer. Math. Soc. vol. 7 (1960) p. 884, replace the third sentence by "As a corollary, it follows that there exist positive

constants A_1, A_2 such that $A_1 < Q(m,n)/mn < A_2$ for m,n sufficiently large." (Received October 18, 1961.)

61T-289. W. MLAK, University of Maryland, College Park, Maryland. Remarks on mean value theorems in linear spaces.

The note contains an extension of mean value theorems introduced by T. Wazewski. The typical (but not the strongest) result is the following theorem. Let E be a locally convex space and $x(A)$ a countably additive and weakly absolutely continuous set function. Let $Z \subseteq E$ be convex and closed. If the weak derivative $Dx(p) \in Z$ for almost all $p \in U$ then $x(V) \in \text{mes}(V)Z$ for every measurable $V \subset U$. In the proof the Mazur-Eidelheit separation theorems are essentially used. As an application there are proved some mean value theorems of type proved for B -spaces by Aziz and Diaz. (Received October 19, 1961.)

61T-290. J. H. BRAMBLE and L. E. PAYNE, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland. Some integral inequalities for uniformly elliptic operators.

Let u be a sufficiently smooth function defined in an N -dimensional region R with boundary C . Let L denote the uniformly elliptic operator $L(u) = (a^{ij}u_{,i})_{,j}$ where a^{ij} is a piecewise continuously differentiable positive definite matrix. The following L_p bound is obtained ($p \geq 2$):
 (1) $\int_R |u|^p dV \leq 2^{p-1} \{ [p/2K(p)] \int_C a^{-1} \partial u / \partial \nu + |u|^p ds + [p/2K(p)]^p \int_R |L(u)|^p dV \}$ where a is any positive function defined on C , $\partial / \partial \nu$ denotes the conormal derivative and $K(p)$ is a computable bound for the first eigenvalue in an associated elastically supported membrane problem. We also obtain the following inequalities: (2) $\int_C a |u| ds \leq \int_R |L(u)| dV + \int_C |\partial u / \partial \nu + au| ds$ and (3) $\int_C |\partial u / \partial \nu + au| ds \leq \int_R |L(u)| dV + 2 \int_C |\partial u / \partial \nu + au| ds$. (Received October 25, 1961.)

61T-291. P. V. REICHELDERFER, 300 University Hall, 216 North Oval Drive, The Ohio State University, Columbus 10, Ohio. A transformation theory for topological space.

Let T be a continuous transformation from a connected, locally connected, locally compact, separable Hausdorff measure space $\{S, M, u\}$ onto a Hausdorff measure space $\{S', M', u'\}$ having a countable base of open sets whose boundaries are of u' measure zero. Assume that each component C of the inverse under T of every point in S' is compact. Let D be generic for a domain in S which has the property that if it meets a continuum C then it contains C . Using Abstract 61T-267 (Notices Amer. Math. Soc. vol. 8 (1961) no. 6) and familiar restrictions on the measures, assuming that $W'(s', D)$ is a weight function which is integrable u and that WD is its integral with respect to u over S' , then necessary and sufficient conditions are given in order that there exist a real valued function f on S which is integrable u and such that the integral of f over D is WD . When these conditions are satisfied it follows that if H' is any real-valued function on S' which is measurable M' then $H'(s')W'(s', D)$ is integrable u' over S' if and only if $H'T(s)f(s)$ is integrable u over D ; if these functions are integrable their integrals are equal. (Received October 26, 1961.)

61T-292. E. T. PARKER, Remington Rand UNIVAC, Division of Sperry Rand Corporation, Univac Park, St. Paul 16, Minnesota. Computer searching for orthogonal latin squares of order 10.

(1) The author has run an exhaustive search for orthogonal mates of 22 random latin squares of order 10. This was done on UNIVAC[®] 1206 Military Computer; see Abstract 564-71 (Notices Amer. Math. Soc. vol. 6 (1959) p. 798). These latin squares are among a sample of 100 generated on the same computer by R. T. Ostrowski, assisted by K. D. Van Duren (Math. Comput. vol. 15 (1961) p. 295). The subset of the sample was selected by use of a random digit table. Of the 22 latin squares, 7, 8, 6, and 1 have respectively none, one, two and three distinct orthogonal mates. This seems effectively to refute the notion that pairs of orthogonal 10-squares are rare. On the other hand, none of these 22 latin squares extends to a triple of mutually orthogonal latin squares. (2) In a fairly systematic fashion, latin 10-squares from the author's counter-examples of 1959 to Euler's conjecture have been similarly run, their orthogonal mates, and some a further level removed. No triples of orthogonal latin 10-squares have been produced in over forty cases in this category. Running times ranged from 28 to 45 minutes per square. (Received October 27, 1961.)

61T-293. A. H. SMITH, Long Beach State College, Long Beach, California. The index semi-group of a topological space.

Two spaces may have the same Homology and Homotopy groups and still not be topologically equivalent (e.g. the real line and the Euclidean plane). The object is to associate with each space X a semigroup $S(X)$ such that (i) $S(X)$ is a topological invariant, and (ii) for any two spaces X_1 and X_2 if $S(X_1)$ is isomorphic with $S(X_2)$ then X_1 is homeomorphic with X_2 . A characterization is given of the class of semigroups which represent spaces and a method is provided for finding Čech Homology of any given space directly from the algebraic structure of the semigroup representing the space. The semigroup $S(X)$ is constructed as follows: Let X be any T_1 -space (i.e. a topological space in which every point is a closed set). Define $S(X)$ to be the class of all closed sets in X . Define multiplication by $b \cdot c = b \cap c$ (intersection) for every pair of elements $b, c \in S(X)$. The algebraic structure of $S(X)$ alone is sufficient to reconstruct a homeomorphic image of X . (Received October 30, 1961.)

61T-294. R. H. SORGENFREY and M. P. BERRI, University of California, 405 Hilgard Avenue, Los Angeles 24, California. Minimal regular spaces.

The present investigation was inspired by a 1941 paper of N. Bourbaki in which minimal Hausdorff topologies were studied. The following similar results are established for minimal regular topologies, i.e., those for which no strictly weaker ($=$ smaller) regular topologies exist: (1) A regular space is minimal regular if and only if each open filter base which is equivalent to a closed filter base and which has a unique adherent point converges (to that point). (2) A minimal regular completely regular space is compact. (3) There exists a noncompact minimal regular space. (Received October 30, 1961.)

61T-295. R. W. CARROLL, Rutgers, The State University, New Brunswick, New Jersey. Some singular quasi-linear Cauchy problems.

Let $L = \partial^2/\partial t^2 + [\alpha(t) + \beta(t)]\partial/\partial t + \gamma(t)$ and consider the Cauchy problem $L\bar{w} + Q(t)\Lambda\bar{w} + f(t, \bar{w}) = 0$, $\bar{w}(\tau) = T \in D(\Lambda)$, $\bar{w}_t(\tau) = 0$, $t \rightarrow \bar{w}(t) \in \mathcal{E}^0(D(\Lambda))$, $t \rightarrow \bar{w}(t) \in \mathcal{E}^2(\mathcal{H})$, $0 \leq \tau \leq t \leq b < \infty$, where Λ is a self-adjoint operator, semi-bounded below, with dense domain $D(\Lambda)$ in a separable Hilbert space $\mathcal{H}$. The hypotheses on α, β, γ, Q are as in Abstract 573-31, Notices Amer. Math. Soc. vol. 7 (1960) p. 875; in particular α is singular at $t = 0$ and Λ may be assumed positive and invertible with closed range $R(\Lambda)$. Suppose for example that $(\xi, v) \rightarrow \Lambda f(\xi, \Lambda^{-1}v): R \times R(\Lambda) \rightarrow R(\Lambda)$ is continuous, $\|\Lambda f(\xi, \Lambda^{-1}v) - \Lambda f(\xi, \Lambda^{-1}u)\| \leq k_1(\xi)\omega_1(\|v - u\|)$ with $k_1 \in L^1$ and ω_1 Osgoodian, and $\|\Lambda f(\xi, \Lambda^{-1}v)\| \leq k_2(\xi)\omega_2(\|v\|)$ with $k_2 \in L^1$ and ω_2 Wintnerian. Then following Foias, Gussi, and Poenaru, Acad. R. P. Romine (1958) pp. 283-304 it is shown that the above problem has at least one solution which will be unique if $\|f(t, u) - f(t, v)\| \leq k_3(t)\omega_3(\|u - v\|)$ with $k_3 \in L^1$ and ω_3 Osgoodian. Variations in the hypotheses and results are studied. (Received October 31, 1961.)

61T-296. F. J. KOSIER, University of Wisconsin, Madison 6, Wisconsin. On certain stable algebras of degree two.

Let A be a simple power-associative over an algebraically closed field F of characteristic $p \neq 2, 3, 5$ satisfying the identity $(1) x(xy) + (yx)x = 2(xy)x$. (Algebras satisfying this (1) were introduced by the author in a paper to appear in Trans. Amer. Math. Soc.) Then A has a unity element 1 and we say that A has degree two if $1 = u + v$ where u and v are primitive orthogonal idempotents of A . It is known that A is the vector space direct sum of the subspaces $A_u(\lambda)$, $\lambda = 0, 1, 2$; where $x \in A_u(\lambda)$ if and only if $ux + xu = \lambda x$. A is stable, if for every idempotent u of A , $A_u(\lambda)A_u(1) + A_u(1)A_u(\lambda) \subseteq A_u(1)$, $\lambda = 0, 2$. Suppose now that A is stable of degree two. Then, using the methods of Oehmke (Proc. Amer. Soc. vol. 12 (1961) pp. 151-158), it is shown that A is a flexible J -simple algebra. But then A also satisfies $[x, [x, y]] = 0$ and thus belongs to a class of quadratic algebras defined by Schafer (Abstract 578-45 Notices Amer. Math. Soc. vol. 8 (1961) p. 153) so that his results apply. (Received November 1, 1961.)

61T-297. FRANK KOSIER, University of Wisconsin, Madison 6, Wisconsin. On certain non-associative algebras.

Let A be a power-associative algebra over a field F of characteristic $p \neq 2, 3, 5$. This paper defines $(x, y, z) = (xy)z - x(yz)$ and notes that A is associative if and only if $(x, y, z) = 0$ for all $x, y, z \in A$. It would then seem natural to study algebras which satisfy an identity of the form I. $(x, y, z) = +(\sigma(x), \sigma(y), \sigma(z))$ for all $x, y, z \in A$ where σ is some permutation of the three elements. Indeed, many such algebras have been studied and these include the right alternative $((x, y, z) = -(x, z, y))$, the left alternative $((x, y, z) = -(y, x, z))$, and the flexible $((x, y, z) = -(z, y, x))$. There are eleven nontrivial identities of the form I. Then suppose these are enumerated so that (11) $(x, y, z) = (z, y, x)$. A is defined to be semi-simple whenever it has nil radical zero. Theorem 1. If A is semi-simple and satisfies some one of the identities (1) - (10), then A has a unity element and is the direct sum of simple algebras. Theorem 2. If A is simple then A is one of the following: (i) a Jordan algebra; (ii) a quasi-associative

algebra; (iii) an algebra of degree one or two. These results do not hold for algebras satisfying (11). (For the structure of these algebras see a forthcoming paper of the author to appear in Trans. Amer. Math. Soc.) (Received November 2, 1961.)

61T-298. J. P. BRANNEN, 800 East 23, Austin 5, Texas. Concerning Hausdorff transformations and absolutely convergent sequences. Preliminary report.

Suppose A is an infinite complex triangular matrix; $A_{n,p} = 0$ if $p > n$. The following two statements are equivalent: (i) if $\{b_n\}_{n=0}^{\infty}$ is an absolutely convergent complex number sequence, i.e. $\sum_{p=1}^{\infty} |b_p - b_{p-1}|$ converges, then the sequence $\{\sum_{p=0}^n A_{n,p} b_p\}_{n=0}^{\infty}$ converges and (ii) (a) there exists a number K such that $|\sum_{p=0}^j A_{n,p}| < K$; $j \leq n = 0, 1, 2, \dots$, (b) the sequence $\{\sum_{p=0}^n A_{n,p}\}_{n=0}^{\infty}$ converges, and (c) the sequence $\{A_{n,p}\}_{n=p}^{\infty}$ converges for $p = 0, 1, 2, \dots$. Let $R[0,1]$ denote the function set such that f belongs to it only in case f is Riemann integrable on $[0,1]$. For each f in $R[0,1]$, let M_f denote the point set such that $P: (x,y)$ belongs to it only in case P is a point of the graph of f , x is in $[0,1]$, and f is continuous at x . If f is in $R[0,1]$ and M_f has only one limit point on the Y -axis, then the Hausdorff matrix $[H,f]$ satisfies the conditions of statement (ii). There exist functions f and g in $R[0,1]$ such that each of M_f and M_g has two limit points on the Y -axis; the matrix $[H,f]$ satisfies the conditions of statement (ii) and the matrix $[H,g]$ does not. (Received November 2, 1961.)

61T-299. A. H. COPELAND, JR, Northwestern University, Evanston, Illinois. Isology functors. Preliminary report.

An isology theory is simply a homology theory, on a category of embeddings (= homeomorphisms into), in which the Homotopy Axiom has been modified by replacing the word "homotopy" by "isotopy". There are isology theories which distinguish various contractible spaces. In fact, given a system of abelian groups G_{pq} (p and q positive integers), there is an isology theory in which the p th isology group of the q -dimensional cube I^q is G_{pq} . If X is a combinatorial n -manifold and if $\{T_r\}$ is an isology theory, then there exists a spectral sequence whose terms E_{pq}^2 are the (ordinary) homology groups $H_p(X; T_q(I^n))$ and whose limit terms yield the isology groups of X . There is an isology theory $\{T_r\}$ such that for 1-complexes X and Y , an embedding $f: X \rightarrow Y$ is an isotopy equivalence if and only if $f_*: T_r(X) \rightarrow T_r(Y)$ is an isomorphism for all r . (Received November 3, 1961.)

61T-300. D. M. TOPPING, Tulane University, New Orleans 18, Louisiana. Spaces of summable functions. Preliminary report.

Recently an axiomatic description of the space of all summable functions was given (D. Topping, Lebesgue spaces of summable functions, Proc. Amer. Math. Soc. vol. 12 (1961) pp. 773-777). An example in the above paper shows that $\mathcal{L}^1(X,m)$, where (X,m) is a finite measure space, satisfies the axioms for an $(\mathcal{AL})$ -space. The set Λ of linear lattice functionals chosen there fails to satisfy Axiom 4 if m is not finite (consider the summable sequences). To circumvent this difficulty, the ring $\mathcal{S}$ of measurable sets of finite measure is employed. The linear lattice functionals appear as two-valued finitely additive measures on $\mathcal{S}$; $v(\lambda)$ denotes the nonzero value of λ . Λ is then taken to be the set of all linear lattice functionals λ with $v(\lambda) \leq 1$. The algebra $B = \{f \in L^1: \|f\| = \sup |\lambda(f)| < \infty;$

$\lambda \in \Lambda$ clearly contains the simple functions on $\mathcal{S}$ and it is easily seen to be complete in the supremum norm. Since B turns out to be boundedly σ -complete, its maximal ideal space is basically disconnected (every cozero set has open closure). Thus any integration theory over an "abstract" measure space is isomorphic to one over a locally compact basically disconnected space. (Received November 3, 1961.)

61T-301. J. A. WOLF, The Institute for Advanced Study, Princeton, New Jersey. Discrete groups, symmetric spaces, and global holonomy.

Let M be a connected simply connected (CSC) Riemannian symmetric space, n the maximum dimension of a flat totally geodesic submanifold (FTGS) which is CSC, D a discontinuous group of isometries with M/D compact, and A an abelian subgroup (AS) of D . Then A preserves a FTGS (say S_A) whose image in M/D is compact, $A = \text{finite} \times F$ with F free abelian on $m \leq n$ generators, S_A can be taken to be a maximal FTGS if $m = n$, and D has a free AS on n generators; if M/D is a manifold, then it has a maximal FTGS which is closed, and its holonomy group is compact (finitely many components). The bounds come from estimates of the "size" of an AS A of a discrete uniform subgroup D of a Lie group $L = E \times G$ where G is reductive with G/G_0 finite, K is a compact group of automorphisms of a vector group V and $E = K \cdot V$. Techniques of L. Auslander show that $G' = D \cdot (V \times G)$ is reductive with G'/G'_0 finite, so the estimates need only be made for the case $L = G$. The main tool of the paper (an element of G is semisimple if and only if some power preserves a geodesic on a certain symmetric space) shows that D consists of semisimple elements of G . A result of Borel-Mostow gives a Cartan subgroup H of G normalized by A . Now $A/(A \cap H)$ is finite and one has the obvious bounds on the "size" of $A \cap H$. (Received October 13, 1961.)

61T-302. KATSUMI NOMIZU, Brown University, Providence 12, Rhode Island. Local homogeneity of Riemannian manifolds.

A Riemannian manifold is said to be locally homogeneous if for any two points x and y there is an isometry of a neighborhood of x onto a neighborhood of y which maps x upon y . M is infinitesimally homogeneous if for any x and y there is an isometric linear isomorphism between the tangent spaces at x and y which preserves the curvature tensor and all its successive covariant differentials. By using a result in [On local and global existence of Killing vector fields, Ann of Math, vol. 72 (1960) pp. 105-120, presented to the Annual Meeting, January 1960] and a lemma of A. Nijenhuis [A theorem on sequences of local affine collineations and isometries, Nieuw Arch. Wisk. vol. 2 (1954), pp. 118-125], it is proved that infinitesimal homogeneity implies local homogeneity. By analyzing the essential argument in the Annals paper, one can also obtain a local version of a theorem of Singer [Infinitesimally homogeneous spaces, Comm. Pure Appl. Math, vol. 13 (1960) pp. 685-697]. The local version of this nature is more general, since local homogeneity implies global homogeneity when M is assumed to be simply connected and complete. (Received November 7, 1961.)

61T-303. KATSUMI NOMIZU, Brown University, Providence 12, Rhode Island. Lie algebra of Killing generators.

Concerning the set of Killing generators $k(x)$ at each point of a Riemannian manifold M defined in the paper [On local and global existence of Killing vector fields, Ann. of Math. vol. 72 (1960) pp. 105-120], it is now proved that $k(x)$ actually forms a Lie algebra with respect to the bracket operation defined in $g(x)$, while $g(x)$ itself is not a Lie algebra in general. (This settles the question left in the Annals paper and corrects the somewhat confused statement there.) The subset $h(x)$ consisting of all elements of the form $(0, A) \in k(x)$ forms a subalgebra of $k(x)$. The natural positive definite inner product in $k(x)$ defined by $((X, A), (Y, B)) = g(X, Y) - \text{trace } AB$ is invariant by $\text{ad}(h(x))$ so that the Lie algebra $k(x)$ is the direct sum (as vector space) $k(x) = m(x) + h(x)$, where $m(x)$ is the orthogonal complement of $h(x)$ and $[m(x), h(x)] \subset m(x)$. Some results in the Annals paper can be reformulated in terms of the subspace $m(x)$. For example, if x is k -regular and if $\dim m(x) = \dim M$, then x has a neighborhood which is locally homogeneous (see the above abstract). The remaining question is: if $\dim m(x) = \dim M$ at every point, is $\dim k(x)$ constant on M ? (Received November 7, 1961.)

61T-304. AZRIEL ROSENFELD, 90 Laurel Hill Terrace, New York 33, New York. Generalized composition series and the unification of introductory group theory.

A fundamental concept in the study of algebraic structures is the introduction of natural "derived" structures on the subsets, image (or quotient) sets, and factor (= "opposite" of product) sets of the underlying set of a given structure. In algebraic structures which are "at least groups", there are natural correspondences between these derived structures and certain successively more specialized kinds of subsets of the underlying set, which may be called stable (including stability under inverses, where applicable), normal, and supplemented subsets, respectively. Nine kinds of series analogous to the classical "composition series" can be defined by replacing neither, either, or both occurrences of the word "normal" in the classical definition by "stable" or by "supplemented". For groups, it turns out that the existence of one or another of these kinds of series is equivalent to finite dimensionality, supersolvability, semisimplicity, the existence of a Remak decomposition, etc. The study of these basic themes of group theory can thus be given a strongly unified motivation. (Received November 3, 1961.)

61T-305. A. A. MULLIN, University of Illinois, Urbana, Illinois. On the mutant cosets of any group.

For terminology refer to the author's earlier abstract (Abstract 60T-34, Notices Amer. Math. Soc. vol. 7 (1960) p. 1005) or his forthcoming paper in the Notre Dame Journal of Formal Logic. Theorem. Let $(G, *)$ be any group. Let λ be any integer ≥ 2 . Then in $(G, *)$ every coset (modulo a subgroup $(H, *)$) excepting both H and (λ, H) -mutant cosets (modulo $(H, *)$) is a $(\lambda + 1, G)$ -mutant set. Corollary 1. In the terminology of an earlier paper (Amer. Math. Monthly vol. 68 (1961) p. 487; erratum, in line 3 of definition 1.3 read " M_i " for " $\bigcap_{i \in I} M_i$ ") the only nonmutant coset of a group is the subgroup itself. Corollary 2. The only idempotent element of a group is its identity element. (Received November 8, 1961.)

61T-306. E. E. LAZERSON, Institute for Defense Analyses, Von Neumann Hall, Princeton, New Jersey. Primitive modules and division rings.

By an (A, A) -module M (A an associative ring) is meant an additive abelian group M which is both a left and right A -module subject to the associativity condition $a(mb) = (am)b$ for all $a, b \in A$ and $m \in M$. If in addition M is a left or right simple faithful module for A , M is called a primitive A -module. In this short note an elementary proof is given of the following theorem. An associative ring A possesses a primitive module if and only if A is a division ring. As a corollary one gets Jacobson's theorem that commutative primitive rings are fields. (Received November 10, 1961.)

61T-307. E. E. LAZERSON, Institute for Defense Analyses, Von Neumann Hall, Princeton, New Jersey. The Jacobson radical of a polynomial ring.

Let A be an associative ring, σ an endomorphism of A , and D a σ -derivation, $D: A \rightarrow A$. Let $A[x; \sigma, D]$ be the set of formal polynomials, $\sum a_i x^i$, where $a_i \in A$ and x (is in general) a noncommuting indeterminate over A . If addition of polynomials is defined componentwise and multiplication is effected by using the rule $xa = \sigma(a)x + D(a)$ (all $a \in A$) and distributivity, then $A[x; \sigma, D]$ forms a ring. This paper proves: If A is commutative and σ is an automorphism of finite order satisfying $\sigma D = D\sigma$, then the Jacobson radical, $J(A[x; \sigma, D])$, of $A[x; \sigma, D]$ is $M[x; \sigma, D]$ (where M is the maximal nil ideal of A invariant under σ and D). This result contains Snapper's theorem on the radical of ordinary polynomial rings as a special case. An idea of Amitsur leads to $J(A[x; \sigma, D]) \subseteq M[x; \sigma, D]$ while the reverse inclusion depends on a combinatorial argument. Further investigations of $J(A[x; \sigma, D])$ are made with relaxed restrictions on A , σ , and D . Finally the inclusion relationships between the various radicals of $A[x; \sigma, D]$ are discussed. (Received November 10, 1961.)

61T-308. E. E. LAZERSON, Institute for Defense Analyses, Von Neumann Hall, Princeton, New Jersey. Derivations and nil rings.

Let A be a commutative, torsion free, associative ring and D a derivation, $D: A \rightarrow A$. If M is the maximal nil ideal of A , then $D(M) \subseteq M$. Suppose $a \in M$, $a \neq 0$, and $a^p = 0$ (for some integer $p > 1$) with $a^{p-1} \neq 0$. We prove: The subring of M generated by the set $\{D^i(a) | i = 0, \dots, n\}$ has index of nilpotency at most $(n+1)(p-1)+1$ where n is an arbitrary non-negative integer. The proof is based on work of Howard Levi. An example is given that indicates this result is best possible for all n, p . (Received November 10, 1961.)

61T-309. L. CARLITZ, Duke University, Durham, North Carolina. Binomial coefficients in an algebraic number field.

Let $K = R(\theta)$ denote an algebraic number field of degree n over the rationals; let $\mathfrak{P}$ be a prime ideal of K and p the rational prime divisible by $\mathfrak{P}$. Let $K_{\mathfrak{P}}$ denote the set of numbers of K that are integral $(\text{mod } \mathfrak{P})$. It is proved that the binomial coefficients $C_{a,m}$ are integral $(\text{mod } \mathfrak{P})$ for all a in $K_{\mathfrak{P}}$ and all $m \geq 1$ if and only if $\mathfrak{P}$ is a prime ideal of the first degree and moreover p does not divide the discriminant of K . As a corollary if $K_{\mathfrak{P}}$ denotes the set of numbers of K that are integral $(\text{mod } p)$

then the binomial coefficients $\binom{a}{m}$ are integral (mod p) for all a in K_p and all $m \geq 1$ if and only if $(p) = \mathfrak{P}_1 \mathfrak{P}_2 \dots \mathfrak{P}_n$, where the $\mathfrak{P}_r$ are distinct prime ideals (of the first degree) of K . (Received November 10, 1961.)

61T-310. L. CARLITZ, Duke University, Durham, North Carolina. Summation of some series of Bessel functions.

Rutgers (Nederl. Akad. Wetensch. Indag. Math. vol. 45 (1942) pp. 9290936, 987-993) has summed a great many series of the type $\sum_{n=0}^{\infty} (-1)^n (v+2n)^k J_{v+2n}(x)$. In the present paper Rutgers' results are generalized and put in a more explicit form. Also a connection with certain functions of importance in finite differences is indicated. (Received November 10, 1961.)

61T-311. L. CARLITZ, Duke University, Durham, North Carolina. The coefficients in the expansion of certain products.

Explicit formulas are obtained for the coefficients A_{rs} , B_{rs} defined by $\prod_{i=1}^{\infty} (1 - p^i x - q^i y)^{-1} = \sum A_{rs} x^r y^s$, $\prod_{i=1}^{\infty} (1 - p^i x - q^i y) = \sum B_{rs} x^r y^s$. Similar but more complicated results are obtained when the number of parameters is arbitrary. (Received November 10, 1961.)

61T-312. L. CARLITZ, Duke University, Durham, North Carolina. An interpolation problem.

The problem of explicitly evaluating the series $\sum_{n=0}^{\infty} (-1)^n (v+2n)^{2k+1} J_{v+2n}(x)$ gave rise to the problem of determining the coefficients $c_{k,s}$ in the representation $x^{2k} = \sum c_{k,2s-1} (x - \alpha - s + 1)_{2s-1} \cdot (x + \alpha - s + 1)_{2s-1} + \sum c_{k,2s} (x - \alpha - s + 1)_{2s} (x + \alpha - s)_{2s}$, where $(x)_s = x(x+1) \dots (x+s-1)$ and α is arbitrary. The case α an integer or half an odd integer requires special treatment. (Received November 10, 1961.)

61T-313. L. CARLITZ, Duke University, Durham, North Carolina. Generating functions for powers of certain sequences of integers.

J. Riordan (in a paper to appear in the Duke Math. J.) has found a recurrence formula for the function $f_k(x) = \sum_{n=0}^{\infty} f_n^k x^n$, where $f_n = f_{n-1} + f_{n-2}$, $f_0 = f_1 = 1$. In the present paper a similar result is found for $u_k(x) = \sum_{n=0}^{\infty} u_n^k x^n$, where $u_n = pu_{n-1} - qu_{n-2}$, $u_0 = 1$, $u_1 = p$, where p, q are arbitrary parameters. Also a comparison formula is obtained for $v_k(x) = \sum_{n=0}^{\infty} v_n^k x^n$. In addition two mixed recurrences containing both $u_k(x)$ and $v_k(x)$ are obtained. (Received November 10, 1961.)

61T-314. M. A. HYMAN, IBM Systems Center, 7220 Wisconsin Avenue, Bethesda 14, Maryland. Signature analysis.

The "signature" of a source is a pattern (or set of patterns) containing information by which the source may be detected and distinguished from other sources. "Coding" is the process by which the source impresses its identity on the pattern; "decoding" is the process by which the key information is extracted from the signature so that the source can be recognized. For a particular type of coding,

specification of a relatively few "characters" serve to distinguish the source from other members of the class being considered. Given a class of signatures, one can distinguish three problems. The "characterization" problem involves finding the type of coding used and the values of the characters for each source; one also determines the systematic and statistical variation of these characters. Having solved the characterization problem, one can examine a given signal and ask: "Does it contain a signature out of the class considered?" ("detection" problem). Alternatively, "Is source S present?" ("identification" problem). Characterization, which often requires extensive and sophisticated data processing, can be done a priori. Detection and identification, which usually must be done under operational conditions, fortunately require relatively simple data processing. Various types of coding are considered here, with attention to techniques of characterization, detection, and identification. Applications suggest themselves - including passive and active sonar, radar, seismology, and electrocardiography. (Received November 10, 1961.)

61T-315. J. V. RYFF, Stanford University, Stanford, California. On vector-valued functions. Preliminary report.

The notion of a weak derivative of a vector-valued function from a real interval to a Banach space gives rise to the possibility of weak differential equations. Existence proofs do not seem to have been considered. A proof is given for the equation $x' = \phi(t; x)$ considered exclusively in the weak topology in the case that x belongs to a reflexive Banach space. Uniqueness is also discussed. Alexiewicz (Studia Math, vol. 11 (1950) pp. 185-196) has shown that weakly differentiable vector-valued functions are strongly differentiable almost everywhere. There is an analogous question with differentiability replaced by continuity. It is shown that weak continuity implies strong except possibly on a set of first category. (Received November 3, 1961.)

61T-316. K. W. KWUN, 210 North Hall, University of Wisconsin, Madison, Wisconsin. On 3-manifolds that are not simply connected.

Theorem. A connected 3-manifold X is simply connected if and only if each arc in X lies in a simply connected neighborhood. Suppose X is not simply connected. Then there exists a polygonal simple closed curve L in X that does not shrink to a point. Let A be a Cantor set of Antoine constructed around L . Then the arc containing A does not lie in any simply connected neighborhood. For this conclusion the following fact is used: Let R be a 3-cube containing two linked simple closed polygons C_1 and C_2 in its interior. Let R be imbedded in a 3-manifold X . If U is a simply connected open subset of X that contains C_1 and C_2 , then C_1 and C_2 can be joined by an arc that lies in $U \cdot \text{Int } R$. (Received October 30, 1961.)

61T-317. K. W. KWUN, 210 North Hall, University of Wisconsin, Madison, Wisconsin. Upper semicontinuous decompositions of the n -sphere.

A decomposition of a space X is a collection of disjoint subsets of X whose sum is X . An I-set in S^n is any set homeomorphic to the complement of a countable subset of S^n . Given a set F in S^n , $G(F)$ denotes the decomposition of S^n consisting of the components of F and the points of $S^n - F$.

Theorem. A decomposition G of S^n is (1) upper semicontinuous, (2) has only a countable number of nondegenerate elements and (3) has the n -sphere decomposition space if and only if $G = G(F)$ for some 1-set F in S^n . ($n > 1$). Corollary. Let A and B be two countable subsets of S^n . There exists a homeomorphism h of S^n into itself such that $h(A) = B$ if and only if $S^n - A$ and $S^n - B$ are homeomorphic. ($n > 1$). Remarks. The theorem and the corollary are false for $n = 1$. Also, one cannot replace the words countable sets in the corollary by arcs. Theorem. Let G be an upper semicontinuous decomposition of S^n . Let F be the sum of the nondegenerate elements of G . Suppose for each $\epsilon > 0$, there exist a finite number of n -cells $D_1, \dots, D_k$ such that $F \subset \sum \text{Int } D_i$ and each $\text{Int } D_i$ is an ϵ -neighborhood of some element of G . Then the decomposition space is an n -sphere. (Received November 2, 1961.)

61T-318. K. W. KWUN, 210 North Hall, University of Wisconsin, Madison, Wisconsin.
A consequence of a theorem of Andrews and Curtis.

Bing (Ann. of Math. vol. 70) showed the existence of nonmanifold cartesian factor of E^4 . Rosen showed (Ann. of Math. vol. 73) that such factor can be really bad. However, their spaces were nevertheless homotopy manifolds. This fact follows from the fundamental theorem (Proc. Amer. Math. Soc. vol. 12) of Kwun. Curtis was the first to show (Abstract 571-15) Notices, Amer. Math. Soc. vol. 7 (1960) p. 482) the existence of a nonhomotopy manifold which is a cartesian factor of E^4 . Using the recent result of Andrews and Curtis: N -space modulo an arc, the following result is obtained. Theorem. For each $n \geq 3$, there exists a generalized manifold X such that $X \times E = E^{n+1}$ but no open subset U of X is a homotopy n -manifold. (Hence, U is not an open n -cell.) In choosing arcs to be used, the arc of Blankinship (Ann. of Math. vol. 53) is first slightly modified. (Received November 2, 1961.)

61T-319. R. K. GETOOR, University of Washington, Seattle 5, Washington. The asymptotic distribution of the number of zero free intervals of a stable process.

Let $\{X(t); t \geq 0\}$ be the one dimensional symmetric stable process of index α , $0 < \alpha \leq 2$. We assume $X(0) = 0$ and that the sample functions are normalized to be right continuous and have left hand limits everywhere. Let $Z(\omega) = \{t: 0 \leq t \leq 1, X(t, \omega) = 0 \text{ or } X(t-, \omega) = 0\}$, then $Z(\omega)$ is a closed subset of $[0, 1]$ for almost all ω . As such its complement in $[0, 1]$ is a countable union of disjoint (relatively) open intervals. Let $N_\alpha(\epsilon)$ be number of such intervals which exceed ϵ in length. It is known that $Z(\omega) = \{0\}$ for almost all ω if $0 < \alpha \leq 1$. In the present paper we prove that if $1 < \alpha \leq 2$, then for each $x > 0$, $\lim_{\epsilon \downarrow 0} P[\Gamma(1/\alpha)\epsilon^{1-1/\alpha} N_\alpha(\epsilon) \leq x] = F_\alpha(x)$, where $F_\alpha(x)$ is a Mittag-Leffler distribution function on $x \geq 0$ which is uniquely determined by its moments $\int_0^\infty x^k dF_\alpha(x) = k! \{\Gamma[1 + k(1 - 1/\alpha)]\}^{-1}$, $k = 0, 1, \dots$. (Received November 6, 1961.)

61T-320. L. CARLITZ, Duke University, Durham, North Carolina. Some arithmetic properties of a special sequence of polynomials in the Gaussian field.

Kelisky (Duke Math. J. vol. 26 (1959) pp. 569-581) has defined a set of rational integers T_n by means of $\sum T_n x^n/n! = \exp(\arctan x)$. More generally Carlitz (Duke Math. J. vol. 26 (1959) pp. 583-590) has defined the polynomial $T_n(z)$ by means of $\sum T_n(z) x^n/n! = \exp(z \arctan x)$. It was proved in particular that if $a + bi$ is a number of the Gaussian field that is integral (mod p), where p is a prime, then $T_p(a + bi) \equiv 0$ or $2a \pmod{p}$ according as $p \equiv 1$ or $3 \pmod{4}$. In the present paper the residue of $T_p(a + bi) \pmod{p^2}$ is determined when $p \equiv 1 \pmod{4}$. When $p \equiv 3 \pmod{4}$ the residues of $T_p(a)$, $T_p(bi) \pmod{p^2}$ are determined. (Received November 10, 1961.)

61T-321. DANIEL GORENSTEIN, Clark University, Worcester, Massachusetts, and R. I. SANDLER, Institute for Defense Analyses, Princeton, New Jersey. On almost-commuting permutations.

If A and B are two permutations in the symmetric group on a finite set X which commute on most points of X , we consider the question of how closely B can be approximated by an element in the centralizer of A . We give examples which show that, except when all, or almost all, the cycles of A are of the same length n , not much can be said in general about how well B can be so approximated. Thus, the bulk of the paper is devoted to the case when A is a product of cycles of the same length n , and in this case we obtain upper and lower bounds for the degree of approximation which depend only upon n and the number of fixed points of the commutator $[A, B]$, but not on the structure of B . (Received November 16, 1961.)

61T-322. FRANK KOSIER, University of Wisconsin, Madison 6, Wisconsin. On a generalization of alternative rings. Preliminary report.

In the following A is assumed to be a power-associative ring of characteristic $p \neq 2$ which satisfies (1) $(x, x, y) = (y, x, x)$ where $(x, y, z) = (xy)z - x(yz)$. Clearly (1) holds in any alternative ring. Identity (1) is also a generalization of (2) $(x, y, z) = (z, y, x)$. (For examples and results on rings satisfying (2) see a forthcoming paper of the author to appear in Trans. Amer. Math. Soc.) Let e be an idempotent of A . Then A has a Peirce decomposition as $A = A_{11} + A_{10} + A_{01} + A_{00}$ where $x \in A_{ij}$ if and only if $ex = ix$ and $xe = jx$. Moreover, these A_{ij} multiply just as in the alternative case with the possible exceptions that the A_{ii} need not be subrings. Using this decomposition the author proves the following: Theorem. Let A be a simple power-associative ring of characteristic $p \neq 2$ which satisfies (1). Suppose further that A has an idempotent e such that $A_{10} + A_{01} \neq 0$. Then A is either a simple associative ring or a Cayley-Dickson algebra over its center. This is essentially the best result one can expect; for the author has constructed simple (non-nil) finite dimensional power-associative algebras over an arbitrary field of characteristic $p \neq 2$ which satisfy (1) but do not possess identity elements. (Received November 13, 1961.)

61T-323. A. A. MULLIN, University of Illinois, Urbana, Illinois. On doubly mutant cosets generated by a skew field.

Theorem. Let $(R, +, \cdot)$ be any skew field (with multiplicative identity 1). Let $(I, +, \cdot)$ be one of its left (or right) ideals. Consider the cosets R/I of the additive group $(R, +)$. Define addition $+$ of the cosets as the usual sum of sets induced by the additive composition law of $(R, +)$. Define multiplication $*$ of the cosets by the relation $(a + I) * (b + I) = (a \cdot b) + I$, where $a \in R$ and $b \in R$. Then every coset of R/I , except I , is an additive $(2, R)$ -mutant set of $(R, +, \cdot)$ and every coset of R/I , except both I and $(1 + I)$, is a doubly $(2, R)$ -mutant set, i.e., a $(2, R)$ -mutant set with respect to both $+$ and $*$.

Corollary. In a skew field the additive identity element is the only doubly idempotent element and 1 is the only other idempotent element. (Received November 15, 1961.)

61T-324. R. SRINIVASAN, Wayne State University, Detroit 2, Michigan. On Lusin probability spaces.

D. Blackwell (Proceedings of the Third Berkeley Symposium, vol. 2) has introduced a class of probability spaces which is more restricted than the class of perfect probability spaces introduced by Gnedenko and Kolmogorov. The concept introduced is that of a Lusin space; that is, a measurable space $(\Omega, \mathcal{M})$ such that (i) $\mathcal{M}$ is separable, and (ii) the range of every real-valued $\mathcal{M}$ -measurable function f on Ω is an analytic set. The purpose of the note is to prove the following: Theorem. If $(\Omega, \mathcal{M})$ is a Lusin space, then any probability measure μ defined on $(\Omega, \mathcal{M})$ is compact. It follows from this result that Lusin spaces are always perfect; for if μ is a compact probability measure of a probability space $(\Omega, \mathcal{M}, \mu)$, then $(\Omega, \mathcal{M}, \mu)$ is quasi-compact, or, equivalently, perfect. (Received November 15, 1961.)

61T-325. SMBAT ABIAN, University of Pennsylvania, Philadelphia 4, Pennsylvania, and A. B. BROWN, Queens College of the City University of New York, Flushing 67, New York. Fixed points of continuous mappings into Euclidean n -space.

Let K^n be an n -complex in Euclidean n -space R^n whose n -cells are positively oriented and f a continuous mapping from K^n into R^n all of whose fixed points are isolated and inner points of the n -cells of K^n . Moreover, let the order of a fixed point of f be defined (following Brouwer) as the degree of the mapping on a direction sphere determined by the directions $p, f(p)$ for p on a sufficiently small positively oriented $(n - 1)$ -sphere with center at the fixed point. Furthermore, let the turning index of the boundary of K^n be defined as the degree of such a mapping of the boundary on the direction sphere. Theorem. The algebraic sum of the orders of the fixed points of K^n under f equals the turning index of the boundary of K^n under f . Example. Let K^3 be a solid torus in R^3 and p an inner point of K^3 . Let f map K^3 continuously into R^3 carrying every boundary point of K^3 radially away from p (or, in any direction but radially toward p). Then, since the turning index of the boundary is $+1$, there must be a fixed point. (Received November 10, 1961.)

61T-326. SMBAT ABIAN, University of Pennsylvania, Philadelphia 4, Pennsylvania, and S. B. BROWN, Queens College of the City University of New York, Flushing 67, New York. Fixed points and covering under continuous mappings of a spherical shell.

Let K^n denote the positively oriented homeomorphic image in a Euclidean n -space R^n of the closed region bounded by an $(n-1)$ -sphere S_1^{n-1} and a concentric smaller $(n-1)$ -sphere S_2^{n-1} , where the homeomorphism can be extended to the entire interior of S_1^{n-1} . Let the respective $(n-1)$ -cycles of the boundary of K^n be denoted by σ_1^{n-1} and σ_2^{n-1} . Finally, let f be a continuous mapping of K^n into R^n . Theorem 1. If for no point s_1 of σ_1^{n-1} is $f(s_1)$ outside of σ_1^{n-1} and if for no point s_2 of σ_2^{n-1} is $f(s_2)$ inside σ_2^{n-1} , and if, moreover, for no point p inside σ_2^{n-1} is the index of p relative to $f(\sigma_2^{n-1})$ equal to $(-1)^{n+1}$, then f has at least one fixed point. Theorem 2. If $f(\sigma_1^{n-1})$ and $f(\sigma_2^{n-1})$ are on K^n and if for some point p inside σ_2^{n-1} the index of p relative to $f(\sigma_1^{n-1})$ is not equal to $(-1)^n$, then either f has a fixed point on K^n or $f(K^n)$ covers the entire interior of σ_2^{n-1} . (Received November 10, 1961.)

61T-327. AUBERT DAIGNEAULT, University of Montreal, Montreal, P. Q., Canada. Generalized ultrapowers. Preliminary report.

For notation and terminology see Abstract 587-136. An algebra A is said to be strongly rich if for all $p \in A$ and $i \in I$ there is a term t of A such that $\exists_i p = S(t/i) p$. C is strongly rich. If A is strongly rich so is every extension $A(L)$. Representations of free rich extensions $C(K)$ lead to the following definition. If X , W and K are sets we denote by $F(W^K, X)$ the set of all functions from $H = W^K$ to X which depend on only finitely many elements of K . If now X is a relational system and D is an ultrafilter of the field $S(H)$ of all subsets of H , the subsystem $F(W^K, X)/D$ of the ultrapower X^H/D is called a generalized ultrapower (g.u.). Theorem 1. A g.u. of X is an elementary extension of X . Theorem 2. If X is a complete relational system then any elementary extension of X is isomorphic to a g.u. of X with $\bar{W} = \bar{X}$. Theorem 3. If Y_1 and Y_2 are relational systems of the same type and if $\bar{Y}_1 = \bar{Y}_2 = \omega_\alpha$ then Y_1 and Y_2 are elementarily equivalent iff for some set K and some ultrafilter D of $S(\omega_\alpha^K)$, $F(\omega_\alpha^K, Y_1)/D$ and $F(\omega_\alpha^K, Y_2)/D$ are isomorphic. For similar results see abstracts by S. B. Kochen (559-97 and 576-199) and H. J. Keisler (559-138, 574-35 and 61T-37). The continuum hypothesis is not used here. (Received August 15, 1961.)

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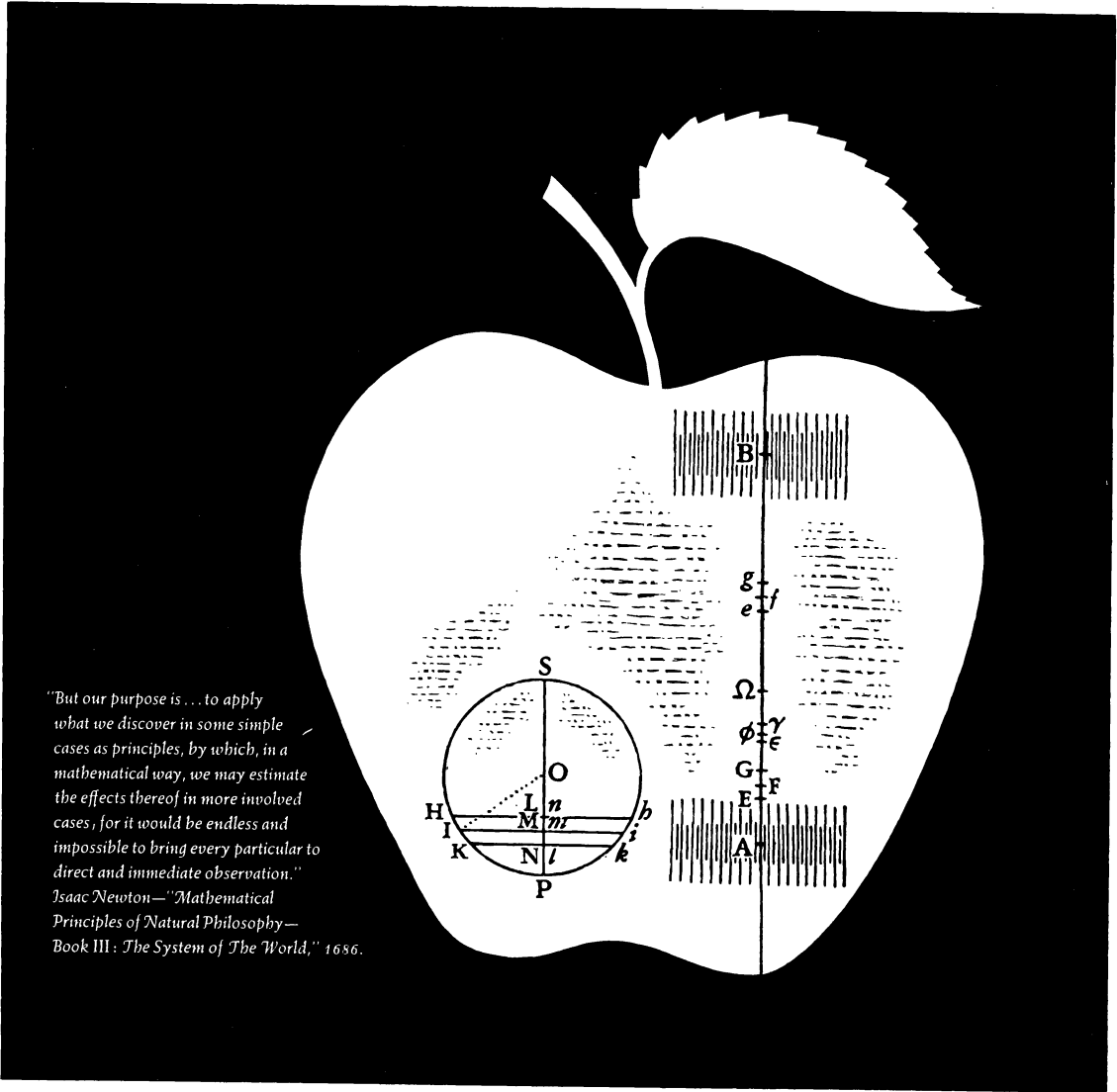
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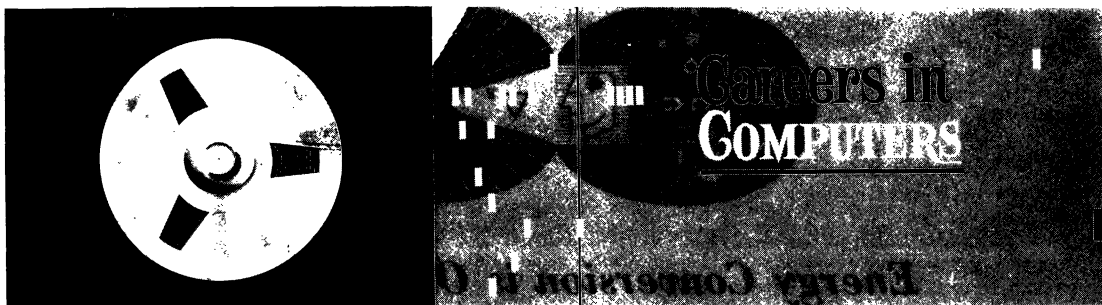
MATHEMATICAL PROGRAMMERS — Analog

To apply analog computer programming and simulation techniques to the solution of problems in advanced missile systems analysis, on EASE computer. These positions require a BS or MS with one of the following majors: Mathematics, Physics, Electrical or Mechanical Engineering, plus at least one years' experience in analog computer programming or applied mathematics.

- **IF** the problems and challenges inherent in some of today's most advanced aerospace development programs interests you . . .
- **THEN** contact immediately:

**Mr. R. P. Bucci, Personnel Department 220
Chrysler Missile Division
P. O. Box 2628, Detroit 31, Michigan**

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Ph. D.'s in Mathematics, Statistics, Applied Mathematics or E. E. with interests in any of the following areas:

- to perform studies of the concepts of system availability from the viewpoint of on line data processing systems; knowledge of systems engineering or probability theory necessary.
- to develop new analysis techniques required in the design of programming large real time computer systems; broad technical background required with specialization in Statistics or Modern Applied Mathematics.
- to work on the exploration of new techniques in fault location on systems including new logic, diagnostic programming techniques and applied programming techniques.
- to work on new technology integration; to explore and develop new methods of performing control logic buffering arithmetic, etc.; also integrate methods in new technology such as solid logic and thin films.
- to work on the design of complex integrated systems; minimum two years previous experience in product design and development of electronic or electro-mechanical system components; design and operational experience with computer systems and components is highly desirable.
- to work on digital and/or analog computers, solution of circuit design and analysis problems; strong background in Applied Mathematics desired.
- to work on digital computer solutions of magnetic circuit and magnetic device analysis problems.
- to work on digital computer solutions of statistical circuit design and analysis problems; strong statistical and electronic engineering background desired.
- to participate in the planning and execution of a full line of computer systems, plan programming systems to accompany the new machines, or the logic design and engineering of one or more of the new systems.....to \$25,000

Creative Programmers — Professionals who wish to participate in advancing the state of the art, with experience or training in:

- | | |
|----------------------------------|-------------------------|
| Compiler Writing | System Design |
| Automatic Programming | Language Analysis |
| Artificial Language Construction | Information Retrieval |
| Non Numerical Mathematics | Artificial Intelligence |
| Symbolic Manipulation | Operations Research |
| Game Playing | Symbolic Logic |
| List Processing Techniques |to \$25,000 |

Programmers

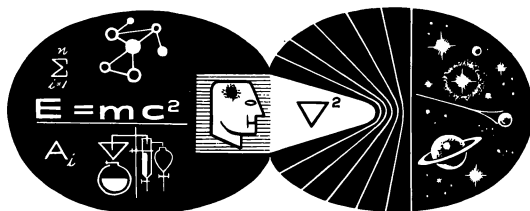
Experience to include programming on large scale equipment with programs and applications of a Business, Scientific or Engineering nature. 7090 experience preferred.....to \$13,000

Our client, a leader in the data processing field (non military areas) is presently expanding a new division (East Coast location). All positions are fee paid. Professionals whose interests and qualifications are in the above areas, submit complete resume with salary requirements to:

DATAMAN*
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PERSONNEL CONSULTANTS
 6 Beacon St., Boston, Mass. - Richmond 2-1932

Contact Arthur J. Girdis during American Mathematical Society Annual Meeting in Cincinnati, Ohio, January 22-26 at the Sheraton-Gibson Hotel

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MS WITH MINIMUM OF 4 YEARS EXPERIENCE in the field of engineering and math. Use mathematical modeling to set up trade-off studies for rocket engine systems and components in the areas of performance, weight, etc., and provide product planning information.

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MS or MA in MATHEMATICS OR SCIENCE with extensive knowledge of probability and statistics. Plan and

co-ordinate analyses, advise on use of analytical methods, coordinate operations research activities, etc. 2 years experience desirable in reliability, process control, design analysis, or operations research.

MS IN MATHEMATICS, SCIENCE OR ENGINEERING with knowledge of probability and statistics. To design and analyze experimental data and interpret the results, determine the optimum conditions, etc. Experience desirable in reliability, process control or design analysis.

MS IN MATHEMATICS to set up equations of motion for rocket and guidance systems. Simulate computer and determine system stability and performance. Extensive background in math desirable.

If you are qualified and interested in becoming a part of the GM-Allison team, we will welcome an interview. Send your resume today to: Mr. V. A. Rhodes, Professional and Scientific Placement, Dept. 401, Allison Division, General Motors Corporation, Indianapolis 6, Indiana.

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