

ON THE MOTION OF A DROP DUE TO A SURFACE TENSION GRADIENT

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1. INTRODUCTION

The motion at low Reynolds number of a drop in a quiescent unbounded liquid due to a surface tension gradient acting on the interface of the two liquids is investigated theoretically by means of a singular - perturbation solution of the axisymmetric equations of motion. Thus, the motion is originated in an interface in a system of two liquids that are not in a compositional equilibrium. We suppose that the quantity of surfactant which contaminates this interface is much smaller than the volume of drop. Special attention is paid to the deformation of the drop. It is shown that for small values of Weber number, the deformation becomes proportioned to the square of the values of the surface tension gradient. Mathematically, this deformation is apparent by taking into account inertia terms in Navier-Stokes equations.

2. THE STATEMENT OF THE PROBLEM

The motion of a viscous liquid drop immersed into an immiscible liquid when the interface between the two liquids is contaminated by a surfactant was investigated experimentally and, up to some extent, theoretically in [1] and [2]. Thus we refer to these papers for experimental picture and also for experimental results. Unfortunately in both, the drop is considered undeformable. The same simplification is operated in our work [3]. In spite of a large number of works devoted to motion of drops (see for example the review article [4]) the dependence of the deformations of a drop on such surface-active materials acting as surface tension gradients is not considered. In this respect, the aim of this note is to determine qualitatively this dependence when no buoyancy force is present (both fluids have the same density ρ).

As a method of investigation we use the analysis from [5]. Thus we consider that the problem at hand is equivalent with that when the drop is fixed and the gradient of surface tension determines the uniform velocity field U at large distance from the drop. So, we propose for U the expression

$$(1) \quad U = \frac{a \cdot \tau}{\mu_s}$$

1991 *Mathematics Subject Classification.* 76D05, 76D45, 35B40.

Key words and phrases. Drop, surface tension, small deformations.

where a is the radius of the drop, τ is the surface tension gradient and μ_s is the viscosity of the interface. With this in mind the analysis from [5] can be used directly.

3. ASYMPTOTIC RESULTS AND DISCUSSIONS

For small Reynolds number, and for moderate values (of order $0(10^2)$) of the ratio of the viscosity of the interior to that of exterior fluid, the most important result in [5] reads as follows

$$(2) \quad \zeta = 0(We),$$

where ζ is the deformation of the drop and $We = \rho a U^2 / \sigma_0$ is the Weber number with σ_0 - a reference surface tension (the deformation ζ is considered smaller than the radius of the drop). Corresponding to the experiments reported in [1] and [2] the Weber number is smaller than unity. Anyway, the asymptotic result (2) implies the fact that the deformation of the drop is actually proportioned with the square of τ . This result seems very plausible.

At least two questions remain open. They are the stability and non-uniqueness of such configurations. In spite of some expert studies like [6] and [7] these problems are essentially unsolved especially when the deformation ζ is not restricted to be small in comparison with the radius of the drop.

Acknowledgments.

The first author wishes to express his gratitude to Professor E. Chifu who has introduced him to this subject and who has given valuable advice.

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